

THE ROOTS AND TANGENTS OF A CUBIC

The main technical problem for this assignment is to be able to plot an accurate graph of a function on a computer. You may wish to try the freely available program `gnuplot`, or the programs `Maple` or `Mathematica`, which are available on the University Computing Services machines. If you are unable to render a plot, you should see me ASAP.

Consider the cubic polynomial $p(x) = x^3 - 8x^2 + 17x - 10$.

1. Find the three real distinct roots a_1, a_2, a_3 of the polynomial p . Label them so that $a_1 < a_2 < a_3$.
2. Plot the graph of p . You will want to use a computer to do this.
3. On your graph of p , draw the tangent line when $x = (a_1 + a_2)/2$. What is the x -intercept of this tangent line?
4. Compute analytically the x -intercept of this tangent line.
5. It is an amazing fact that this has nothing to do with the choice of the two smallest roots. Print another copy of the graph of p , and draw the three tangent lines at the points $(a_1 + a_2)/2$, $(a_2 + a_3)/2$ and $(a_3 + a_1)/2$, and see where they intersect the x -axis.
6. Compute analytically the x -intercepts of the other two tangent lines. Now state a theorem about the x -intercepts of the three lines tangent to the cubic p at the averages of the roots.
7. (Extra for experts). This involves pushing some algebra around. Show that for an arbitrary cubic polynomial with three distinct real roots, the lines tangent to the cubic at the average of two of the roots intersect the cubic at the other root.