Here are a baker's dozen problems for you to practice for the midterm. Points have been given to these problems so that you have some idea of what I consider to be the difficulty. The problems worth ten points are considerably more involved than the others, and so you should not be worried if you can not do them all. However, you should be able to do the questions worth two points each. As always, if you are having difficulties, you need to either see me, your TA, the continuous tutorial, or the others in your assignment group so that things can be resolved *ahead of time.* !!

1. Answer the following true or false (2 points each).
   (a) The set of all values of the slope of the line $y = mx + 2$ for which the x-intercept exceeds $1/2$ is $m > 4$.
   (b) The following limit is -1.5
   $$\lim_{x\to 1} \frac{x^3 - 3x + 2}{x^3 - 5x^2 + 7x - 3}$$
   (c) The equation $\cos x = x$ has at least one solution.
   (d) If f and g are both continuous at 0, then the composite function $f \circ g(x) = f(g(x))$ is continuous at 0.

2. Compute the following derivatives (2 points each)
   (a) $y'(2)$ if $y = \sqrt[3]{x + \sqrt{x}}$.
   (b) $y'(\pi/4)$ if $y = \sin(\tan x)$.
   (c) $y'(1)$ if $y = e^{3x} \ln \sqrt{x}$.

3. The cissoid of Diocles is the curve given by $y^2(2 - x) = x^3$. The normal line to a curve at the point p is the line passing through p that is perpendicular to the tangent line to the curve at p. Find both the tangent line and the normal line to the cissoid at $(x, y) = (1, 1)$. (5 points)

4. The velocity of a heavy meteorite entering the earth's atmosphere is inversely proportional to $\sqrt{s}$ when it is s kilometers from the earth's center. Show that the meteorite's acceleration is inversely proportional to s^2. (5 points)

5. An alert hiker sees that a boulder falls off a cliff and notices that it falls the last third of the way in 3/2 seconds. What is the height of the cliff? You may assume the boulder falls straight down with an initial velocity of zero, and that the acceleration due to gravity is $10m/s^2$. Hint: I found this easiest to do by considering the inverse function $t = t(s)$. (10 points)

6. If $y = \frac{1}{2}(e^x - e^{-x})$, express x in terms of y. (4 points)

7. If a, b, c are positive numbers, show that

$$\log_a c = \log_a b \cdot \log_b c.$$

(8 points)

8. Two resistors of R_1 and R_2 ohms are connected in parallel in an electric circuit to make an R ohm resistor. The value of R can be found from the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 is decreasing at 1 ohm/sec and R_2 is increasing at a rate of 1/2 ohm/sec at what rate is R changing when $R_1 = 75$ ohms and $R_2 = 50$ ohms? (6 points)

9. A particle moves along $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at a rate of 11 units per second. Find dx/dt when $x = 3$. (10 points)

10. A boat is pulled onto a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 meter higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock? (6 points)

11. A sandbag is dropped from a balloon at a height of 200 feet when the angle of elevation to the sun is 30°. Find the rate at which the shadow of the sandbag is traveling along the ground when the sandbag is at a height of 125 feet. (Hint: The position of the sandbag is given by $s(t) = 200 - 16t^2$.) (10 points)

12. A woman standing on a cliff is watching a sailboat through a telescope as the boat approaches the shoreline directly below her. If the telescope is 80 metres above the water level and the boat is approaching at 8 meters per second, at what rate is the angle of the telescope changing when the boat is 80 meters from shore? (6 points)

13. Let a vertical disc of radius 10 centimeters be lowered into a basin of water at a rate of 2 centimeters per minute (i.e. the height of the center of the disc is decreasing at 2 cm/min.) At what rate is the wetted area of the disc increasing when the center of the disc is 5 centimeters above the surface of the water? (10 points)