1. Find the smallest positive value of the constant m that will make $mx - 1 + (1/x)$ non-negative for all positive x.

2. To find $x = \sqrt[q]{a}$, we apply Newton's method to $f(x) = x^q - a$. Here we assume that a is a positive real number and q is a positive integer. Show that x_1 is a "weighted average" of x_0 and a/x_0^{q-1}, and find coefficients $m_0 > 0, m_1 > 0$ such that

$$x_1 = m_0 x_0 + m_1 \left(\frac{a}{x_0^{q-1}} \right), \qquad m_0 + m_1 = 1.$$

What conclusion would you reach if x_0 and a/x_0^{q-1} were equal? What would be the value of x_1 in that case?

3. Two quantities are in the golden ratio φ if the ratio between the sum of those quantities and the larger one is the same as the ratio between the larger one and the smaller. At least since the Renaissance, many artists and architects have proportioned their works to approximate the golden ratio – especially in the form of the golden rectangle, in which the ratio of the longer side to the shorter is the golden ratio – believing this proportion to be aesthetically pleasing. Thus given two numbers a and b in the golden ratio φ, we have

$$\frac{a+b}{a} = \frac{a}{b} = \varphi$$

This equation has as its unique positive solution the algebraic irrational number

$$\varphi = \frac{1+\sqrt{5}}{2}$$

You can show this by first showing that φ is a root of $\varphi^2 - \varphi - 1 = 0$. Use Taylor's theorem for the square root function to find a third order estimate for φ.

4. Given $y = f(x) = x^2$ and the partition $P = \{0, 1/4, 1/2, 3/4, 1\}$, find the upper and lower sums $U(f, P)$ and $L(f, P)$, and hence bounds on the definite integral

$$\int_0^1 f(x)\,dx.$$