UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 251 — L01 Winter 2009

MIDTERM EXAM **A** [March 11, 2009 (Wednesday)]

Time: 50 minutes. PLEASE write your Name on the very last page.
NO CALCULATORS.
Total Marks = 100. Work all problems. Marks are shown in brackets.

Student ID: ____________________

[Marks]

1. Find the natural domain of the function $f(x) = \dfrac{1}{\sqrt{|2x-7|-3}}$. (Write your answer in interval

[10] notations)

2. Evaluate the following limits:

[8] (a) $\lim_{x \to 2} \frac{\sqrt{4x+1} - 3}{x - 2}$;

[8] (b) $\lim_{x \to +\infty} \sqrt{\frac{4x^3 + 2x - 1}{x^3 - 3x + 7}}$

3. Let $f(x)$ be the function defined piecewise by

$$f(x) := \begin{cases} \dfrac{4}{3x} - \dfrac{a}{3}, & \text{if} \quad x < 2 \\ 1, & \text{if} \quad x = 2 \\ \dfrac{x^2 - 1}{x - a}, & \text{if} \quad x > 2 \end{cases}.$$

Write your answer in the space provided (no explanation needed):

[2] (a) The function $f(x)$ is defined at $x = 2$ with $f(2) =$ __________;

[2] (b) $\lim_{x \to 2^-} f(x) =$ __________;

[2] (c) $\lim_{x \to 2^+} f(x) =$ __________;

[3] (d) The function $f(x)$ is continuous at $x = 2$ if $a =$ __________; (Hint: $\lim_{x \to 2} f(x) = f(2)$)

[3] (e) The function $f(x)$ has a removable discontinuity at $x = 2$ if $a =$ __________;

[2] (f) The function $f(x)$ has an infinite discontinuity at $x = 2$ if $a =$ __________;

[2] (h) With the information above, we can conclude that the function is not differentiable at $x = 2$ if $a =$ __________.

4. Use techniques of differentiation to find the derivative (do not simplify) $y'(x) = \frac{dy}{dx}$ for:

[8] (a) $y = \dfrac{x\tan(x)}{x^2+1}$;

[8] (b) $y = \sqrt{(x^2+1)\sin^3(x)}$.

5. Let $f(x) = \sqrt{5 - x}$.

[10] (a) Find the linear approximation of $f(x)$ at the point $x_0 = 1$.

[6] (b) Use the linear approximation from part (a) to estimate the value of $\sqrt{4.12}$.

[2] (c) What is the error in this approximation if $\sqrt{4.12} = 2.029$?

6. Given that $y^2 + xy = x^3$,

[10] (a) Find $y' = \dfrac{dy}{dx}$;

[6] (b) find the equation of the tangent line to the curve $y^2 + xy = x^3$ at the point $(2, 2)$.

[8] 7. Find the (x, y)-coordinates of the point on the graph of the function $f(x) = x^4 + 2x - 1$ where the tangent line is perpendicular to the line $6y + x - 4 = 0$.

Name:	Student ID:	Marks: