NAME $\qquad$

1. Let $f_{1}(x)=x, f_{2}(x)=1 / x, f_{3}(x)=1-x, f_{4}(x)=1 /(1-x), f_{5}(x)=$ $(x-1) / x, f_{6}(x)=x /(x-1)$. Show that the composition of any two functions in the list is again in the list.
2. Let $f(x)=1-(x-1)^{2}$ for $0 \leq x \leq 2$. Sketch $f(x)$. By shifting, sketch $1-f(x+1)$.
3. Given $O=(0,0), A=(a, 0), B=(0, b)$ as the vertices of the triangle $O A B$, find the slope of the line $O P$ if $P$ is the midpoint of the side $A B$.
4. For which $x$ is

$$
\frac{x-1}{x-2} \leq \frac{x+1}{x+2}
$$

5. Prove that the area of any triangle is equal to half the product of any two sides and the sine of the included angle.
6. A wall is $k$ metres high and a second wall, higher than the first, is $m$ metres behind the first one. A ladder must have one end on the ground, pass over the first wall, and reach the second. Express the length of the ladder as a function of the angle that it makes with the ground.
7. Let the equation of a semicircle be $y=\sqrt{r^{2}-x^{2}}, A$ the point $(r, 0)$, $B$ the point $(-r, 0)$, and $C=(c, d)$ any point on the semicircle. Show that the angle $\angle A C B$ is a right angle. Hint: one way to do this is to show that the slopes of two lines are negative reciprocals.
8. (For experts) Show that the area of a triangle with sides of lengths $a, b, c$ is

$$
\sqrt{s(s-a)(s-b)(s-c)}
$$

where $s=(a+b+c) / 2$ is the semiperimeter of the triangle.

