

NAME \_\_\_\_\_

1. Suppose that  $f$  is differentiable and that there are points  $a$  and  $b$  such that  $f(a) = b$  and  $f(b) = a$ . Let  $k(x) = f(f(f(f(x))))$ . Show that  $k'(a) = k'(b)$ .
2. Find the derivative of the following
  - (a)  $y = 2 \tan^2 2x^2$
  - (b)  $y = \tan(\sec x)$
  - (c)  $y = \sqrt{\csc \sqrt{1 + x^2}}$
3. Find the equation of the tangent line to  $y = \tan^2 x$  when  $x = \pi/3$ .
4. For  $f(x) = \sec^2 x$  and  $g(x) = \tan^2 x$ , show that  $f'(x) = g'(x)$ .
5. Suppose that  $f(0) = 0$  and  $f'(0) = 2$ . Calculate the derivative of  $f(f(f(x)))$  when  $x = 0$ .
6. Find  $y'$  by implicit differentiation if

$$\frac{1}{x} + \frac{1}{y} = xy.$$

7. What is  $y'$  if  $\cos xy = x^3 + \tan \sqrt{x^2 + y^2}$ ?
8. What is  $y''$  if  $x^2 + 3xy + 4y^2 + 2x - 7y + 1 = 0$ ?
9. If  $p(x) = (1 + x^9)^{11}$ , compute the one hundredth derivative  $p^{(100)}(x)$ .
10. Find the two points where the curve  $x^2 + xy + y^2 = 7$  crosses the  $x$ -axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?
11. For what value of the constants  $a$ ,  $b$  and  $c$  does the function

$$f(x) = \begin{cases} x^3 & x \leq 1 \\ ax^2 + bx + c & x > 1 \end{cases}$$

have a second derivative at  $x = 1$ ?