- Practice #7
- 1. Using $f(x) = x^2 3$, find the first three approximations for $\sqrt{3}$ by Newton's method starting with $x_0 = 2$. How would you estimate $\sqrt[3]{4}$?
- 2. Compute the limit

$$\lim_{x \to \infty} \left(1 + \frac{1}{x} \right)^x$$

3. Compute the limit

$$\lim_{x \to 0} \frac{9x - 3\sin 3x}{5x^3}$$

4. In L'Hôpital's 1696 textbook, he illustrated his rule by finding the limit

$$\lim_{x \to a} \frac{\sqrt{2a^3x - x^4 - a\sqrt[3]{a^2x}}}{a - \sqrt[4]{ax^3}}$$

where a > 0. Find this limit as well.

- 5. Find the fourth order Taylor approximation to $\ln x$ about x = 1. Use this to estimate $\ln 1.1$
- 6. Find the fourth order Taylor approximation to both $-\ln(1-x)$ and 1/(1-x) about x = 0, and show that you can find the series for 1/(1-x) by differentiating the series for $-\ln(1-x)$.
- 7. Evaluate the following sums

$$\sum_{i=1}^{5} \frac{1}{i} \qquad \sum_{i=0}^{5} 2^{i} \qquad \sum_{n=3}^{6} n!$$

8. Show that the sum of the binomial coefficients satisfy

$$\sum_{k=0}^{n} \left(\begin{array}{c} n\\ k \end{array} \right) = 2^{n}.$$

Hint: use the binomial theorem.