Here are some answers for the problems on the final review list. If you do not know how to solve some of the problems, make sure you talk to someone in the continuous tutorial or your TA or come and see me.

1. (a) False.
   (b) True.
   (c) False.
   (d) 122/9.
   (e) $\ln(e + e^{-1}) - \ln 2$.
   (f) The point of the problem is that you can pull the x out of the integral and then use the product rule and the fundamental theorem of calculus to do the calculation.
   (g) The interval $(-1 - \sqrt{2}, -1 + \sqrt{2})$.
   (h) 1.5/3600 kilometers per second per second.
   (i) By L'Hôpital, the limit is 0.
2. $\sqrt{45}$.
3. The area is 1, and you will no doubt be pleased to know that this is a theorem of Archimedes.
4. This is a straight verification. You only need to remember that $e^* e^{-*} = 1$ for any $*$.
5. Expanding around 16 yields $\sqrt{13} \approx 4 + \frac{1}{8}3 - \frac{1}{256}3^2 + \frac{3}{2048}3^3$.
6. The two numbers are $\frac{1}{4}$ and $35\frac{3}{4}$
7. The line is $(ay - b)/(bx - a) = -1$.
8. Just make sure you only have one inflection point.
9. $(a, b) = (6, 2)$.
10. Construct a continuous function for the time difference and use the intermediate value theorem.
11. $-11\pi/12$ radians per hour.

12. If A is the area, $(r, \phi) = (\sqrt{A}, 2)$.

13. The two lines are $y = \pm x/\sqrt{3}$. It most certainly helps to draw a picture of this problem before you start calculating.

14. If D denotes the distance, then

$$2D\frac{dD}{dt} = 100 + 96\sin(b-a)(\frac{db}{dt} - \frac{da}{dt})$$

with $a = 2\pi/36$, $b = 2\pi/3$, $\frac{db}{dt} = 2\pi$, $\frac{da}{dt} = 2\pi/12$, with a the angle of the hour hand, b the angle of the minute hand.

15. Let S be the surface area. Then the radius and height are $(r, h) = (\sqrt{S/(6\pi)}, \sqrt{2S/(3\pi)})$.

16. 405/4 meters.

17. The derivative of the inverse function is $1/(a-c)^2$.

18. Write

$$\int_1^{ab} \frac{1}{t}\,dt = \int_1^{a} \frac{1}{t}\,dt + \int_a^{ab} \frac{1}{t}\,dt$$

and make the change of variable $u = t/a$ in the last integral.

19. This follows immediately from the definition, as you compute that $fg(-x) = f(-x)g(-x) = -f(x)g(x) = -fg(x)$

20. $y = x^3$ is the most famous example.

21. 4/3.

22. $-5/2$.

23. The derivative is 1.

24. $y = \pm 1$. Just use my all-purpose formula for the tangent line.

25. $x = (-1 \pm \sqrt{29})/2$. The tangents are parallel there.

26. The width W and the depth D are $(W, D) = (12/\sqrt{3}, 12\sqrt{23})$.

27. The function $x - \cos x$ has a non-negative derivative and is negative at $x = 0$ and positive at $x = 2$.

28. This follows by induction with the product rule.

29. This is even easier if you write $y = x^2 + 1/x$.

30. This is for free because you have these already memorized, right?

31. This is straightforward algebra.

32. The algebra becomes simple if you observe that $x = 0$ is a critical point.

33. The line $x = 2$ is a vertical asymptote, and $y = x/2 + 1$ is a slant asymptote.

34. The volume is $16\pi/3$.

35. Let h be the height of the window, $2w$ the width, and P the perimeter. Then $w = 2P/(8 + 3\pi)$.

36. 67 people.

37. The deceleration is $3 \cdot 88^2/(2 \cdot 242)$.

38. The area is two thirds of the base times the height, $A = 2bh/3$.

39. (a) True.
    (b) True.
    (c) True.
    (d) True.
    (e) False.
    (f) False.
    (g) True.

40. You can differentiate the integrals using the fundamental theorem of the calculus.

    (a) $f(4) = 1/2$.
    (b) $f(4) = \sqrt[3]{12}$.

41. The largest area is $1/e$.

42. For $H > 2R$,

$$r = \frac{RH}{2(H - R)},$$

and if $H \leq 2R$,

$$r = R.$$