Here are some problems to try so that you know that you are up to speed for the second quiz.

1. If $f(x) = (x-1)/x$ and $(f \circ g)(x) = x/(x+1)$, what is $g(x)$?
2. Find the following limit if it exists:
$$\lim_{x\to\infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$$
3. Find m and b so that the limit
$$\lim_{x\to\infty} \sqrt{4x^2 + 8x + 42} - (mx + b) = 0$$
The line $y = mx + b$ in this case is called a *slant asymptote* to $\sqrt{4x^2 + 8x + 42}$.
4. Show that the cubic polynomial $p(x) = x^3 - 15x + 1$ has three roots in the interval $[-4, 4]$.
5. Does the parabola $y = 2x^2 - 13x + 5$ have a tangent whose slope is -1? If so, find an equation for the line and the point of tangency. If not, why not?

A few hints and tips. Remember, you must give the problems some effort before you read any hints:

1. Observe that you are being asked to solve $(g(x)-1)/g(x) = x/(x+1)$, which is a linear equation for g.
2. The easiest thing is of course to multiply top and bottom to get rid of the negative exponents.
3. I get the conditions $m^2 = 4$ and $2mb = 8$ in order for the limit to vanish. The easiest way to obtain this is to multiply and divide by the conjugate.

4. If you evaluate the polynomial at enough points (say all the integers in this interval,) you should find enough changes of sign so that you can use the intermediate value theorem on three intervals to get what you want.

5. A little reflection on the derivative of the function shows that you could just as easily solve the problem for an arbitrary value of the slope, say m. In this sense there is nothing special about the value -1.