

Here are some problems to try so that you know that you are up to speed for the third quiz.

1. Compute the derivative of $f(x) = (x + 7)/\sqrt{x^2 + 7}$ when $x = 3$.
2. If $f(x) = x+1$ and $g(x) = x^2$, what is the derivative of $g(f(g(x)))$ when $x = 1$? You should do this two ways. First, use the chain rule, and second, write out the composition completely before differentiating.
3. Consider the function $f(x) = 1/x$, and an interval $[a, b]$ with $a > 0$. Show that the point in the interval where the slope of the tangent to the graph of $y = f(x)$ is equal to the slope of the chord connecting $(a, f(a))$ and $(b, f(b))$ is given by the geometric mean of a and b . Recall that the geometric mean of a and b is equal to \sqrt{ab} . Observe that this point is the point guaranteed to exist by the mean value theorem.
4. Assume that f and g are differentiable on $[a, b]$ and that $f(a) = g(a)$ and $f(b) = g(b)$. Show that there is at least one point between a and b where the tangents to the graphs of f and g are parallel. Draw a picture of this.
5. Check that the point $p = (\sqrt{3}/4, 1/2)$ is on the curve $y^4 = y^2 - x^2$. Solve for $y = y(x)$ explicitly (which you can do by using the quadratic formula twice.) Now find the values of y' and y'' at the point p .

A few hints and tips. Remember, you must give the problems some effort before you read any hints:

1. The only real error to avoid here is to differentiate *before* you substitute the value so that you do not wind up differentiating the constant function.
2. Since this is just differentiating polynomials, there is no problem at all here.

3. This is an easy proof as soon as you realize that you can just calculate both things independently, and only have to push a couple of lines of algebra.
4. One way to do this is to examine the difference function $D(x) = f(x) - g(x)$, and then show that D satisfies the hypotheses of Rolle's theorem.
5. The problem becomes much easier algebraically once you read it carefully enough to realize that you are not required to compute the derivatives using the explicit formula. I would compute the derivatives by differentiating implicitly.