APPENDIX C

Coordinate Planes and Lines

RECTANGULAR COORDINATE SYSTEMS

Just as points on a coordinate line can be associated with real numbers, so points in a plane can be associated with pairs of real numbers by introducing a ***rectangular coordinate system*** (also called a ***Cartesian coordinate system***). A rectangular coordinate system consists of two perpendicular coordinate lines, called ***coordinate axes***, that intersect at their origins. Usually, but not always, one axis is horizontal with its positive direction to the right, and the other is vertical with its positive direction up. The intersection of the axes is called the ***origin*** of the coordinate system.

It is common to call the horizontal axis the ***x-axis*** and the vertical axis the ***y-axis***, in which case the plane and the axes together are referred to as the ***xy-plane*** (Figure C.1). Although labeling the axes with the letters x and y is common, other letters may be more appropriate in specific applications. Figure C.2 shows a uv-plane and a ts-plane—the first letter in the name of the plane always refers to the horizontal axis and the second to the vertical axis.

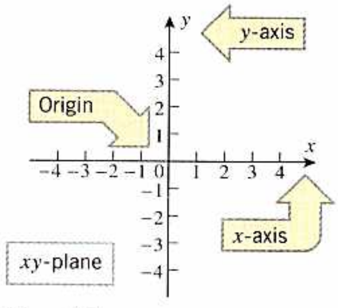


Figure C.1

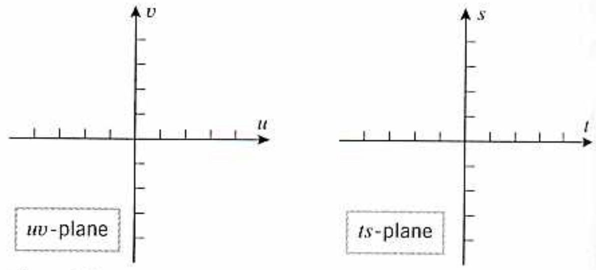


Figure C.2

COORDINATES

Every point P in a coordinate plane can be associated with a unique ordered pair of real numbers by drawing two lines through P, one perpendicular to the x-axis and the other perpendicular to the y-axis (Figure C.3). If the first line intersects the x-axis at the point with coordinate a and the second line intersects the y-axis at the point with coordinate b, then we associate the ordered pair of real numbers (a, b) with the point P. The number a is called the ***x-coordinate*** or ***abscissa*** of P and the number b is called the ***y-coordinate*** or ***ordinate*** of P. We will say that P has ***coordinates*** (a, b) and write $P(a, b)$ when we want to emphasize that the coordinates of P are (a, b). We can also reverse the above procedure and find the point P associated with the coordinates (a, b) by locating the intersection of the dashed lines in Figure C.3. Because of this one-to-one correspondence between coordinates and points, we will sometimes blur the distinction between points and ordered pairs of numbers by talking about the *point* (a, b).

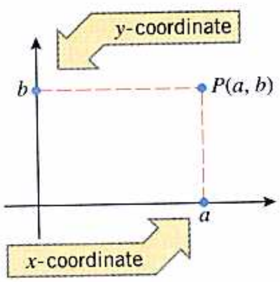


Figure C.3

REMARK. Recall that the symbol (a, b) also denotes the open interval between a and b; the appropriate interpretation will usually be clear from the context.

In a rectangular coordinate system the coordinate axes divide the rest of the plane into four regions called ***quadrants***. These are numbered counterclockwise with roman numerals as shown in Figure C.4. As indicated in that figure, it is easy to determine the quadrant in which a given point lies from the signs of its coordinates: a point with two positive coordinates $(+, +)$ lies in Quadrant I, a point with a negative x-coordinate and a positive y-coordinate $(-, +)$ lies in Quadrant II, and so forth. Points with a zero x-coordinate lie on the y-axis and points with a zero y-coordinate lie on the x-axis.

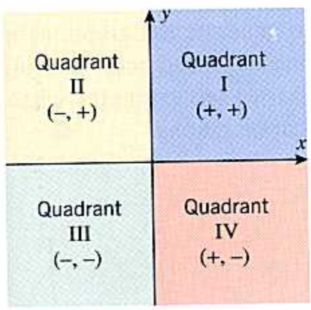


Figure C.4

To ***plot*** a point $P(a, b)$ means to locate the point with coordinates (a, b) in a coordinate plane. For example, in Figure C.5 we have plotted the points

$$P(2, 5), \quad Q(-4, 3), \quad R(-5, -2), \quad \text{and} \quad S(4, -3)$$

Observe how the signs of the coordinates identify the quadrants in which the points lie.

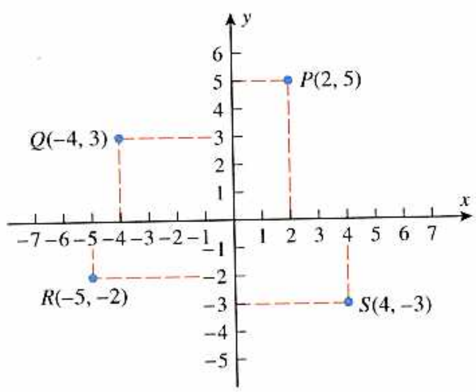


Figure C.5

GRAPHS

The correspondence between points in a plane and ordered pairs of real numbers makes it possible to visualize algebraic equations as geometric curves, and, conversely, to represent geometric curves by algebraic equations. To understand how this is done, suppose that we have an xy-coordinate system and an equation involving two variables x and y, say

$$6x - 4y = 10, \quad y = \sqrt{x}, \quad x = y^3 + 1, \quad \text{or} \quad x^2 + y^2 = 1$$

We define a ***solution*** of such an equation to be any ordered pair of real numbers (a, b) whose coordinates satisfy the equation when we substitute $x = a$ and $y = b$. For example, the ordered pair $(3, 2)$ is a solution of the equation $6x - 4y = 10$, since the equation is satisfied by $x = 3$ and $y = 2$ (verify). However, the ordered pair $(2, 0)$ is not a solution of this equation, since the equation is not satisfied by $x = 2$ and $y = 0$ (verify).

The following definition makes the association between equations in x and y and curves in the xy-plane.

C.1 DEFINITION. The set of all solutions of an equation in x and y is called the ***solution set*** of the equation, and the set of all points in the xy-plane whose coordinates are members of the solution set is called the ***graph*** of the equation.

One of the main themes in calculus is to identify the exact shape of a graph. Point plotting is one approach to obtaining a graph, but this method has limitations, as discussed in the following example.

Example 1 Sketch the graph of $y = x^2$.

Solution. The solution set of the equation has infinitely many members, since we can substitute an arbitrary value for x into the right side of $y = x^2$ and compute the associated y to obtain a point (x, y) in the solution set. The fact that the solution set has infinitely many members means that we cannot obtain the *entire* graph of $y = x^2$ by point plotting. However, we can obtain an *approximation* to the graph by plotting some sample members of the solution set and connecting them with a smooth curve, as in Figure C.6. The problem with this method is that we cannot be sure how the graph behaves *between* the plotted points. For example, the curves in Figure C.7 also pass through the plotted points and hence are legitimate candidates for the graph in the absence of additional information. Moreover, even if we use a graphing calculator or a computer program to generate the graph, as in Figure C.8, we have the same problem because graphing technology uses point-plotting algorithms to generate graphs. Indeed, in Section 1.3 of the text we see examples where graphing technology can be fooled into producing grossly inaccurate graphs. ◀

x	$y = x^2$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)
−1	1	(−1, 1)
−2	4	(−2, 4)
−3	9	(−3, 9)

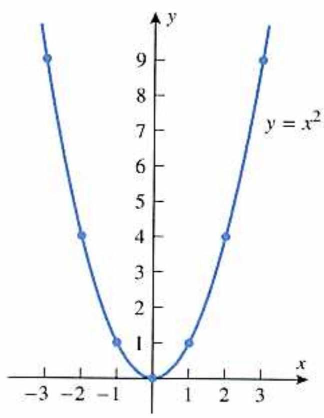


Figure C.6

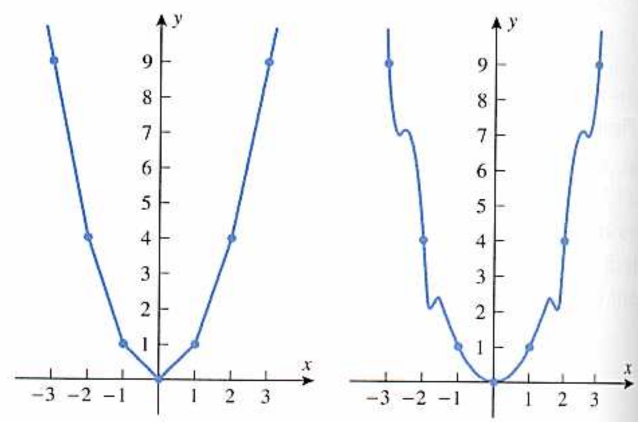
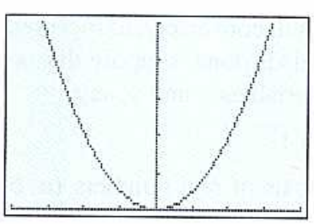


Figure C.7



[−4, 4] × [0, 10]
xScl = 1, yScl = 2

$y = x^2$

Figure C.8

In spite of its limitations, point plotting by hand or with the help of graphing technology can be useful, so here are two more examples.

Example 2 Sketch the graph of $y = \sqrt{x}$.

Solution. If $x < 0$, then $\sqrt{x}$ is an imaginary number. Thus, we can only plot points for which $x \geq 0$, since points in the xy-plane have real coordinates. Figure C.9 shows the graph obtained by point plotting and a graph obtained with a graphing calculator. ◀

Example 3 Sketch the graph of $y^2 - 2y - x = 0$.

Solution. To calculate coordinates of points on the graph of an equation in x and y, it is desirable to have y expressed in terms of x or x in terms of y. In this case it is easier to express x in terms of y, so we rewrite the equation as

$$x = y^2 - 2y$$

Members of the solution set can be obtained from this equation by substituting arbitrary values for y in the right side and computing the associated values of x (Figure C.10). ◀

x	$y = \sqrt{x}$	(x, y)
0	0	$(0, 0)$
1	1	$(1, 1)$
2	$\sqrt{2}$	$(2, \sqrt{2}) \approx (2, 1.4)$
3	$\sqrt{3}$	$(3, \sqrt{3}) \approx (3, 1.7)$
4	2	$(4, 2)$

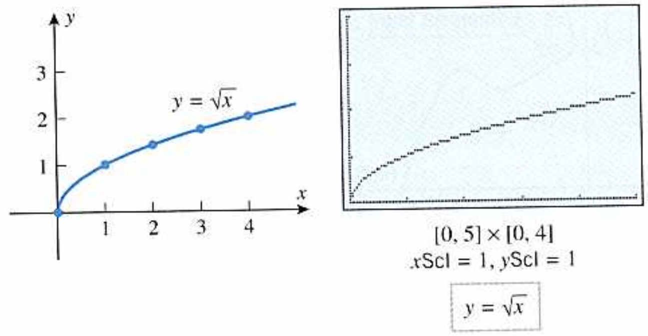
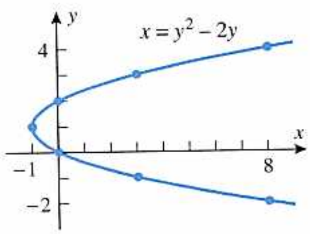


Figure C.9



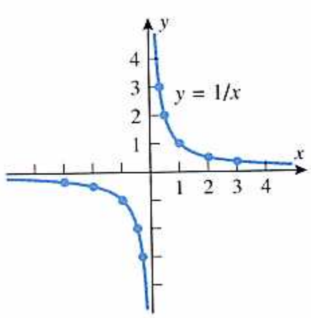
y	$x = y^2 - 2y$	(x, y)
−2	8	$(8, -2)$
−1	3	$(3, -1)$
0	0	$(0, 0)$
1	−1	$(-1, 1)$
2	0	$(0, 2)$
3	3	$(3, 3)$
4	8	$(8, 4)$

Figure C.10

REMARK. Most graphing calculators and computer graphing programs require that y be expressed in terms of x to generate a graph in the xy-plane. In Section 1.8 we discuss a method for circumventing this restriction.

Example 4 Sketch the graph of $y = 1/x$.

Solution. Because $1/x$ is undefined at $x = 0$, we can only plot points for which $x \neq 0$. This forces a break, called a *discontinuity*, in the graph at $x = 0$ (Figure C.11). ◀



x	$y = 1/x$	(x, y)
$\frac{1}{3}$	3	$\left(\frac{1}{3}, 3\right)$
$\frac{1}{2}$	2	$\left(\frac{1}{2}, 2\right)$
1	1	$(1, 1)$
2	$\frac{1}{2}$	$\left(2, \frac{1}{2}\right)$
3	$\frac{1}{3}$	$\left(3, \frac{1}{3}\right)$
$-\frac{1}{3}$	−3	$\left(-\frac{1}{3}, -3\right)$
$-\frac{1}{2}$	−2	$\left(-\frac{1}{2}, -2\right)$
−1	−1	$(-1, -1)$
−2	$-\frac{1}{2}$	$\left(-2, -\frac{1}{2}\right)$
−3	$-\frac{1}{3}$	$\left(-3, -\frac{1}{3}\right)$

Figure C.11

INTERCEPTS

Points where a graph intersects the coordinate axes are of special interest in many problems. As illustrated in Figure C.12, intersections of a graph with the x-axis have the form $(a, 0)$ and intersections with the y-axis have the form $(0, b)$. The number a is called an ***x-intercept*** of the graph and the number b a ***y-intercept***.

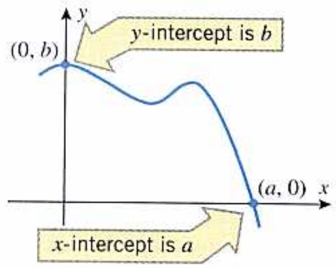


Figure C.12

Example 5 Find all intercepts of

(a) $3x + 2y = 6$ (b) $x = y^2 - 2y$ (c) $y = 1/x$

Solution (a). To find the x-intercepts we set $y = 0$ and solve for x:

$$3x = 6 \quad \text{or} \quad x = 2$$

To find the y-intercepts we set $x = 0$ and solve for y:

$$2y = 6 \quad \text{or} \quad y = 3$$

As we will see later, the graph of $3x + 2y = 6$ is the line shown in Figure C.13.

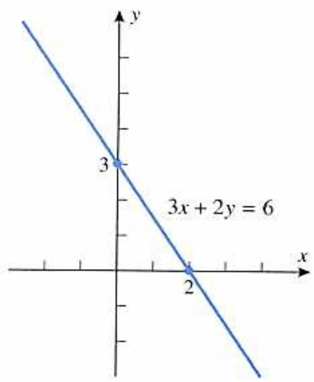


Figure C.13

Solution (b). To find the x-intercepts, set $y = 0$ and solve for x:

$$x = 0$$

Thus, $x = 0$ is the only x-intercept. To find the y-intercepts, set $x = 0$ and solve for y:

$$y^2 - 2y = 0$$

$$y(y - 2) = 0$$

So the y-intercepts are $y = 0$ and $y = 2$. The graph is shown in Figure C.10.

Solution (c). To find the x-intercepts, set $y = 0$:

$$\frac{1}{x} = 0$$

This equation has no solutions (why?), so there are no x-intercepts. To find y-intercepts we would set $x = 0$ and solve for y. But, substituting $x = 0$ leads to a division by zero, which is not allowed, so there are no y-intercepts either. The graph of the equation is shown in Figure C.11. ◀

SLOPE

To obtain equations of lines we will first need to discuss the concept of *slope*, which is a numerical measure of the "steepness" of a line.

Consider a particle moving left to right along a *nonvertical* line from a point $P_1(x_1, y_1)$ to a point $P_2(x_2, y_2)$. As shown in Figure C.14, the particle moves $y_2 - y_1$ units in the y-direction as it travels $x_2 - x_1$ units in the positive x-direction. The vertical change $y_2 - y_1$ is called the ***rise***, and the horizontal change $x_2 - x_1$ the ***run***. The ratio of the rise over the run can be used to measure the steepness of the line, which leads us to the following definition.

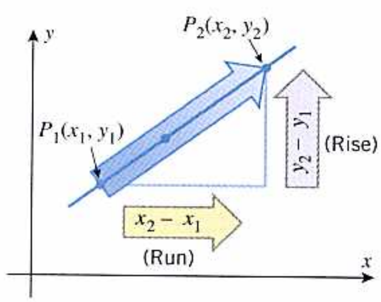


Figure C.14

C.2 DEFINITION. If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are points on a nonvertical line, then the ***slope*** m of the line is defined by

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \tag{1}$$

REMARK. Observe that this definition does not apply to vertical lines. For such lines we have $x_2 = x_1$ (a zero run), which means that the formula for m involves a division by zero. For this reason, the slope of a vertical line is ***undefined***, which is sometimes described informally by stating that a vertical line has ***infinite slope***.

When calculating the slope of a nonvertical line from Formula (1), it does not matter which two points on the line you use for the calculation, as long as they are distinct. This can be proved using Figure C.15 and similar triangles to show that

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2' - y_1'}{x_2' - x_1'}$$

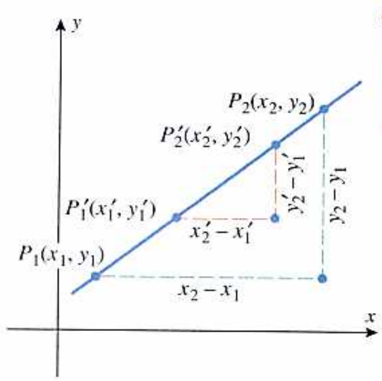


Figure C.15

Moreover, once you choose two points to use for the calculation, it does not matter which one you call P_1 and which one you call P_2 because reversing the points reverses the sign of both the numerator and denominator of (1) and hence has no effect on the ratio.

Example 6 In each part find the slope of the line through

(a) the points (6, 2) and (9, 8)

(b) the points (2, 9) and (4, 3)

(c) the points (−2, 7) and (5, 7).

Solution.

(a) $m = \frac{8-2}{9-6} = \frac{6}{3} = 2$ (b) $m = \frac{3-9}{4-2} = \frac{-6}{2} = -3$ (c) $m = \frac{7-7}{5-(-2)} = 0$ ◀

Example 7 Figure C.16 shows the three lines determined by the points in Example 6 and explains the significance of their slopes. ◀

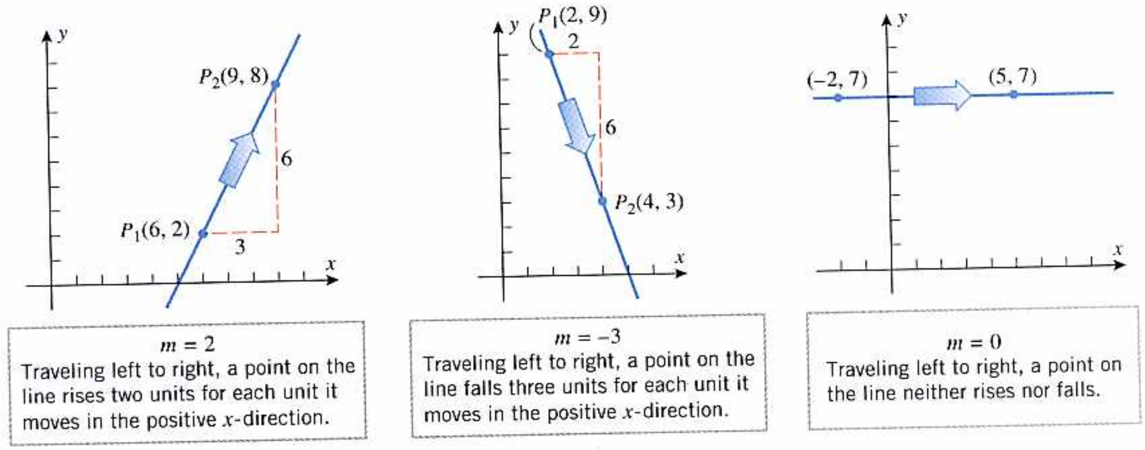


Figure C.16

As illustrated in this example, the slope of a line can be positive, negative, or zero. A positive slope means that the line is inclined upward to the right, a negative slope means that the line is inclined downward to the right, and a zero slope means that the line is horizontal. An undefined slope means that the line is vertical. Figure C.17 shows various lines through the origin with their slopes.

PARALLEL AND PERPENDICULAR LINES

The following theorem shows how slopes can be used to tell whether two lines are parallel or perpendicular.

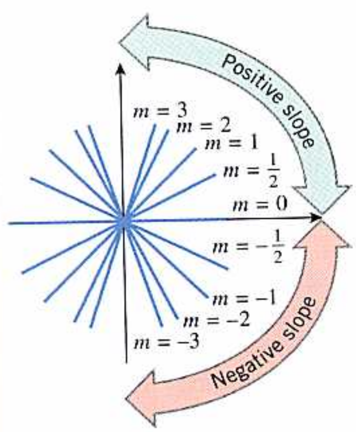


Figure C.17

C.3 THEOREM.

(*a*) *Two nonvertical lines with slopes m_1 and m_2 are parallel if and only if they have the same slope, that is,*

$$m_1 = m_2$$

(*b*) *Two nonvertical lines with slopes m_1 and m_2 are perpendicular if and only if the product of their slopes is −1, that is,*

$$m_1 m_2 = -1$$

This relationship can also be expressed as $m_1 = -1/m_2$ or $m_2 = -1/m_1$, which states that nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of one another.

A complete proof of this theorem is a little tedious, but it is not hard to motivate the results informally. Let us start with part (*a*).

Suppose that L_1 and L_2 are nonvertical parallel lines with slopes m_1 and m_2, respectively. If the lines are parallel to the x-axis, then $m_1 = m_2 = 0$, and we are done. If they are not parallel to the x-axis, then both lines intersect the x-axis; and for simplicity assume that they are oriented as in Figure C.18*a*. On each line choose the point whose run relative to the point of intersection with the x-axis is 1. On line L_1 the corresponding rise will be m_1 and on L_2 it will be m_2. However, because the lines are parallel, the shaded triangles in the figure must be congruent (verify), so $m_1 = m_2$. Conversely, the condition $m_1 = m_2$ can be used to show that the shaded triangles are congruent, from which it follows that the lines make the same angle with the x-axis and hence are parallel (verify).

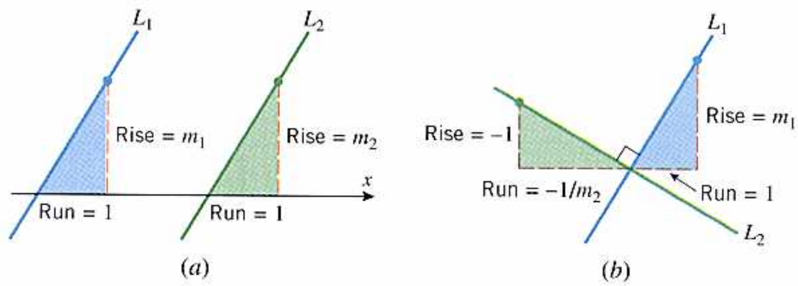


Figure C.18

Now suppose that L_1 and L_2 are nonvertical perpendicular lines with slopes m_1 and m_2, respectively; and for simplicity assume that they are oriented as in Figure C.18*b*. On line L_1 choose the point whose run relative to the point of intersection of the lines is 1, in which case the corresponding rise will be m_1; and on line L_2 choose the point whose rise relative to the point of intersection is -1, in which case the corresponding run will be $-1/m_2$. Because the lines are perpendicular, the shaded triangles in the figure must be congruent (verify), and hence the ratios of corresponding sides of the triangles must be equal. Taking into account that for line L_2 the vertical side of the triangle has length 1 and the horizontal side has length $-1/m_2$ (since m_2 is negative), the congruence of the triangles implies that $m_1/1 = (-1/m_2)/1$ or $m_1 m_2 = -1$. Conversely, the condition $m_1 = -1/m_2$ can be used to show that the shaded triangles are congruent, from which it can be deduced that the lines are perpendicular (verify).

Example 8 Use slopes to show that the points $A(1, 3)$, $B(3, 7)$, and $C(7, 5)$ are vertices of a right triangle.

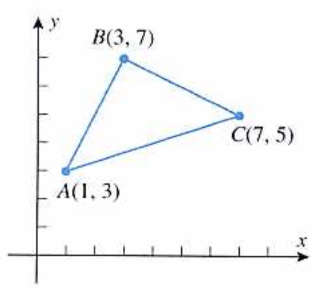


Figure C.19

Solution. We will show that the line through A and B is perpendicular to the line through B and C. The slopes of these lines are

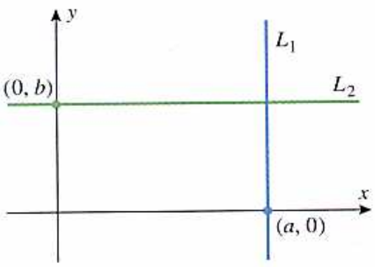
$$m_1 = \frac{7-3}{3-1} = 2 \quad \text{and} \quad m_2 = \frac{5-7}{7-3} = -\frac{1}{2}$$

Slope of the line through A and B — Slope of the line through B and C

Since $m_1 m_2 = -1$, the line through A and B is perpendicular to the line through B and C; thus, ABC is a right triangle (Figure C.19). ◀

LINES PARALLEL TO THE COORDINATE AXES

We now turn to the problem of finding equations of lines that satisfy specified conditions. The simplest cases are lines parallel to the coordinate axes. A line parallel to the y-axis intersects the x-axis at some point $(a, 0)$. This line consists precisely of those points whose x-coordinates equal a (Figure C.20). Similarly, a line parallel to the x-axis intersects the y-axis at some point $(0, b)$. This line consists precisely of those points whose y-coordinates equal b (Figure C.20). Thus, we have the following theorem.



Every point on L_1 has an x-coordinate of a and every point on L_2 has a y-coordinate of b.

Figure C.20

C.4 THEOREM. *The vertical line through $(a, 0)$ and the horizontal line through $(0, b)$ are represented, respectively, by the equations*

$$x = a \quad \textit{and} \quad y = b$$

Example 9 The graph of $x = -5$ is the vertical line through $(-5, 0)$, and the graph of $y = 7$ is the horizontal line through $(0, 7)$ (Figure C.21). ◀

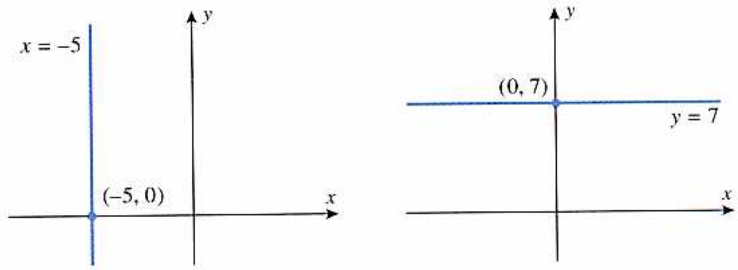
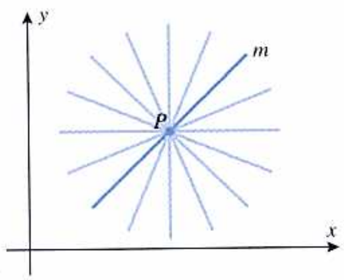


Figure C.21

LINES DETERMINED BY POINT AND SLOPE

There are infinitely many lines that pass through any given point in the plane. However, if we specify the slope of the line in addition to a point on it, then the point and the slope together determine a unique line (Figure C.22).



There is a unique line through P with slope m.

Figure C.22

Let us now consider how to find an equation of a nonvertical line L that passes through a point $P_1(x_1, y_1)$ and has slope m. If $P(x, y)$ is any point on L, different from P_1, then the slope m can be obtained from the points $P(x, y)$ and $P_1(x_1, y_1)$; this gives

$$m = \frac{y - y_1}{x - x_1}$$

which can be rewritten as

$$y - y_1 = m(x - x_1) \tag{2}$$

With the possible exception of (x_1, y_1), we have shown that every point on L satisfies (2). But $x = x_1$, $y = y_1$ satisfies (2), so that all points on L satisfy (2). We leave it as an exercise to show that every point satisfying (2) lies on L.

In summary, we have the following theorem.

C.5 THEOREM. *The line passing through $P_1(x_1, y_1)$ and having slope m is given by the equation*

$$y - y_1 = m(x - x_1) \tag{3}$$

This is called the ***point-slope form*** *of the line.*

Example 10 Find the point-slope form of the line through $(4, -3)$ with slope 5.

Solution. Substituting the values $x_1 = 4$, $y_1 = -3$, and $m = 5$ in (3) yields the point-slope form $y + 3 = 5(x - 4)$. ◀

LINES DETERMINED BY SLOPE AND y-INTERCEPT

A nonvertical line crosses the y-axis at some point $(0, b)$. If we use this point in the point-slope form of its equation, we obtain

$$y - b = m(x - 0)$$

which we can rewrite as $y = mx + b$. To summarize:

> **C.6 THEOREM.** *The line with y-intercept b and slope m is given by the equation*
>
> $$y = mx + b \tag{4}$$
>
> *This is called the **slope-intercept form** of the line.*

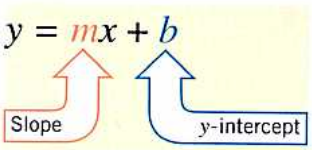


Figure C.23

REMARK. Note that y is alone on one side of Equation (4). When the equation of a line is written in this way the slope of the line and its y-intercept can be determined by inspection of the equation—the slope is the coefficient of x and the y-intercept is the constant term (Figure C.23).

Example 11

EQUATION	SLOPE	y-INTERCEPT
$y = 3x + 7$	$m = 3$	$b = 7$
$y = -x + \frac{1}{2}$	$m = -1$	$b = \frac{1}{2}$
$y = x$	$m = 1$	$b = 0$
$y = \sqrt{2}x - 8$	$m = \sqrt{2}$	$b = -8$
$y = 2$	$m = 0$	$b = 2$

◀

Example 12 Find the slope-intercept form of the equation of the line that satisfies the stated conditions:

(a) slope is -9; crosses the y-axis at $(0, -4)$

(b) slope is 1; passes through the origin

(c) passes through $(5, -1)$; perpendicular to $y = 3x + 4$

(d) passes through $(3, 4)$ and $(2, -5)$.

Solution (a). From the given conditions we have $m = -9$ and $b = -4$, so (4) yields $y = -9x - 4$.

Solution (b). From the given conditions $m = 1$ and the line passes through $(0, 0)$, so $b = 0$. Thus, it follows from (4) that $y = x + 0$ or $y = x$.

Solution (c). The given line has slope 3, so the line to be determined will have slope $m = -\frac{1}{3}$. Substituting this slope and the given point in the point-slope form (3) and then simplifying yields

$$\begin{aligned} y - (-1) &= -\tfrac{1}{3}(x - 5) \\ y &= -\tfrac{1}{3}x + \tfrac{2}{3} \end{aligned}$$

Solution (d). We will first find the point-slope form, then solve for y in terms of x to obtain the slope-intercept form. From the given points the slope of the line is

$$m = \frac{-5 - 4}{2 - 3} = 9$$

We can use either of the given points for (x_1, y_1) in (3). We will use $(3, 4)$. This yields the point-slope form

$$y - 4 = 9(x - 3)$$

Solving for y in terms of x yields the slope-intercept form

$$y = 9x - 23$$

We leave it for the reader to show that the same equation results if $(2, -5)$ rather than $(3, 4)$ is used for (x_1, y_1) in (3). ◀

THE GENERAL EQUATION OF A LINE

An equation that is expressible in the form

$$Ax + By + C = 0 \tag{5}$$

where A, B, and C are constants and A and B are not both zero, is called a ***first-degree equation*** in x and y. For example,

$$4x + 6y - 5 = 0$$

is a first-degree equation in x and y since it has form (5) with

$$A = 4, \quad B = 6, \quad C = -5$$

In fact, all the equations of lines studied in this section are first-degree equations in x and y.

The following theorem states that the first-degree equations in x and y are precisely the equations whose graphs in the xy-plane are straight lines.

> **C.7 THEOREM.** *Every first-degree equation in x and y has a straight line as its graph and, conversely, every straight line can be represented by a first-degree equation in x and y.*

Because of this theorem, (5) is sometimes called the ***general equation*** of a line or a ***linear equation*** in x and y.

Example 13 Graph the equation $3x - 4y + 12 = 0$.

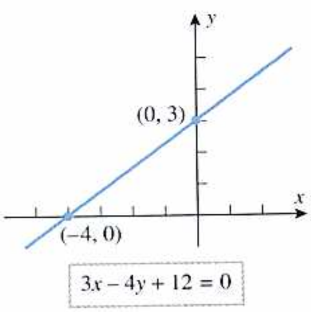


Figure C.24

Solution. Since this is a linear equation in x and y, its graph is a straight line. Thus, to sketch the graph we need only plot any two points on the graph and draw the line through them. It is particularly convenient to plot the points where the line crosses the coordinate axes. These points are $(0, 3)$ and $(-4, 0)$ (verify), so the graph is the line in Figure C.24. ◀

Example 14 Find the slope of the line in Example 13.

Solution. Solving the equation for y yields

$$y = \tfrac{3}{4}x + 3$$

which is the slope-intercept form of the line. Thus, the slope is $m = \frac{3}{4}$. ◀

EXERCISE SET C

1. Draw the rectangle, three of whose vertices are $(6, 1)$, $(-4, 1)$, and $(6, 7)$, and find the coordinates of the fourth vertex.

2. Draw the triangle whose vertices are $(-3, 2)$, $(5, 2)$, and $(4, 3)$, and find its area.

> In Exercises 3 and 4, draw a rectangular coordinate system and sketch the set of points whose coordinates (x, y) satisfy the given conditions.

3. (a) $x = 2$ (b) $y = -3$ (c) $x \geq 0$
   (d) $y = x$ (e) $y \geq x$ (f) $|x| \geq 1$

4. (a) $x = 0$ (b) $y = 0$
   (c) $y < 0$ (d) $x \geq 1$ and $y \leq 2$
   (e) $x = 3$ (f) $|x| = 5$

> In Exercises 5–12, sketch the graph of the equation. (A calculating utility will be helpful in some of these problems.)

5. $y = 4 - x^2$

6. $y = 1 + x^2$

7. $y = \sqrt{x - 4}$

8. $y = -\sqrt{x + 1}$

9. $x^2 - x + y = 0$

10. $x = y^3 - y^2$

11. $x^2 y = 2$

12. $xy = -1$

13. Find the slope of the line through
(a) $(-1, 2)$ and $(3, 4)$ (b) $(5, 3)$ and $(7, 1)$
(c) $(4, \sqrt{2})$ and $(-3, \sqrt{2})$ (d) $(-2, -6)$ and $(-2, 12)$.

14. Find the slopes of the sides of the triangle with vertices $(-1, 2)$, $(6, 5)$, and $(2, 7)$.

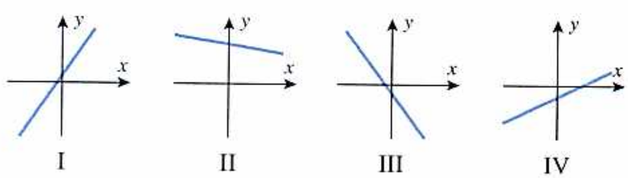
15. Use slopes to determine whether the given points lie on the same line.
(a) $(1, 1)$, $(-2, -5)$, and $(0, -1)$
(b) $(-2, 4)$, $(0, 2)$, and $(1, 5)$

16. Draw the line through $(4, 2)$ with slope
(a) $m = 3$ (b) $m = -2$ (c) $m = -\frac{3}{4}$.

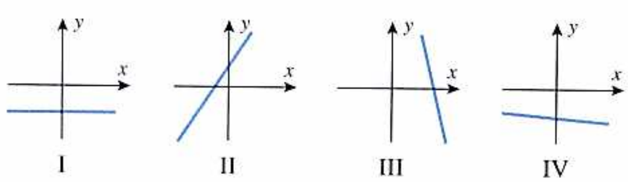
17. Draw the line through $(-1, -2)$ with slope
(a) $m = \frac{3}{5}$ (b) $m = -1$ (c) $m = \sqrt{2}$.

18. An equilateral triangle has one vertex at the origin, another on the x-axis, and the third in the first quadrant. Find the slopes of its sides.

19. List the lines in the accompanying figure in the order of increasing slope.



20. List the lines in the accompanying figure in the order of increasing slope.



21. A particle, initially at $(1, 2)$, moves along a line of slope $m = 3$ to a new position (x, y).
(a) Find y if $x = 5$. (b) Find x if $y = -2$.

22. A particle, initially at $(7, 5)$, moves along a line of slope $m = -2$ to a new position (x, y).
(a) Find y if $x = 9$. (b) Find x if $y = 12$.

23. Let the point $(3, k)$ lie on the line of slope $m = 5$ through $(-2, 4)$; find k.

24. Given that the point $(k, 4)$ is on the line through $(1, 5)$ and $(2, -3)$, find k.

25. Find x if the slope of the line through $(1, 2)$ and $(x, 0)$ is the negative of the slope of the line through $(4, 5)$ and $(x, 0)$.

26. Find x and y if the line through $(0, 0)$ and (x, y) has slope $\frac{1}{2}$, and the line through (x, y) and $(7, 5)$ has slope 2.

27. Use slopes to show that $(3, -1)$, $(6, 4)$, $(-3, 2)$, and $(-6, -3)$ are vertices of a parallelogram.

28. Use slopes to show that $(3, 1)$, $(6, 3)$, and $(2, 9)$ are vertices of a right triangle.

29. Graph the equations
(a) $2x + 5y = 15$ (b) $x = 3$
(c) $y = -2$ (d) $y = 2x - 7$.

30. Graph the equations
(a) $\dfrac{x}{3} - \dfrac{y}{4} = 1$ (b) $x = -8$
(c) $y = 0$ (d) $x = 3y + 2$.

31. Graph the equations
(a) $y = 2x - 1$ (b) $y = 3$
(c) $y = -2x$.

32. Graph the equations
(a) $y = 2 - 3x$ (b) $y = \frac{1}{4}x$
(c) $y = -\sqrt{3}$.

33. Find the slope and y-intercept of
(a) $y = 3x + 2$ (b) $y = 3 - \frac{1}{4}x$
(c) $3x + 5y = 8$ (d) $y = 1$
(e) $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

34. Find the slope and y-intercept of
(a) $y = -4x + 2$ (b) $x = 3y + 2$
(c) $\dfrac{x}{2} + \dfrac{y}{3} = 1$ (d) $y - 3 = 0$
(e) $a_0 x + a_1 y = 0$ $(a_1 \neq 0)$.

In Exercises 35 and 36, use the graph to find the equation of the line in slope-intercept form.

35.

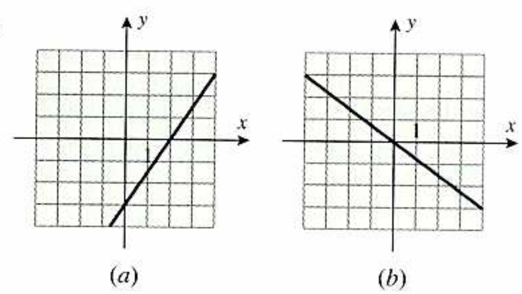


Figure Ex-35

36.

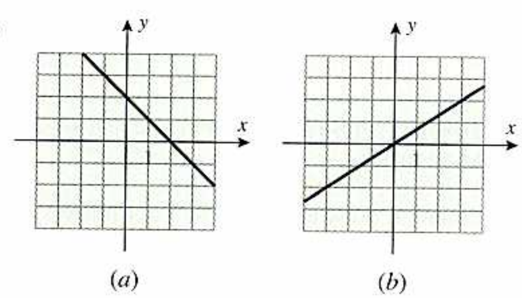


Figure Ex-36

In Exercises 37–48, find the slope-intercept form of the line satisfying the given conditions.

37. Slope $= -2$, y-intercept $= 4$.

38. $m = 5$, $b = -3$.

39. The line is parallel to $y = 4x - 2$ and its y-intercept is 7.

40. The line is parallel to $3x + 2y = 5$ and passes through $(-1, 2)$.

41. The line is perpendicular to $y = 5x + 9$ and its y-intercept is 6.

42. The line is perpendicular to $x - 4y = 7$ and passes through $(3, -4)$.

43. The line passes through $(2, 4)$ and $(1, -7)$.

44. The line passes through $(-3, 6)$ and $(-2, 1)$.

45. The y-intercept is 2 and the x-intercept is -4.

46. The y-intercept is b and the x-intercept is a.

47. The line is perpendicular to the y-axis and passes through $(-4, 1)$.

48. The line is parallel to $y = -5$ and passes through $(-1, -8)$.

49. In each part, classify the lines as parallel, perpendicular, or neither.
(a) $y = 4x - 7$ and $y = 4x + 9$
(b) $y = 2x - 3$ and $y = 7 - \frac{1}{2}x$
(c) $5x - 3y + 6 = 0$ and $10x - 6y + 7 = 0$
(d) $Ax + By + C = 0$ and $Bx - Ay + D = 0$
(e) $y - 2 = 4(x - 3)$ and $y - 7 = \frac{1}{4}(x - 3)$

50. In each part, classify the lines as parallel, perpendicular, or neither.
(a) $y = -5x + 1$ and $y = 3 - 5x$
(b) $y - 1 = 2(x - 3)$ and $y - 4 = -\frac{1}{2}(x + 7)$
(c) $4x + 5y + 7 = 0$ and $5x - 4y + 9 = 0$
(d) $Ax + By + C = 0$ and $Ax + By + D = 0$
(e) $y = \frac{1}{2}x$ and $x = \frac{1}{2}y$

51. For what value of k will the line $3x + ky = 4$
(a) have slope 2
(b) have y-intercept 5
(c) pass through the point $(-2, 4)$
(d) be parallel to the line $2x - 5y = 1$
(e) be perpendicular to the line $4x + 3y = 2$?

52. Sketch the graph of $y^2 = 3x$ and explain how this graph is related to the graphs of $y = \sqrt{3x}$ and $y = -\sqrt{3x}$.

53. Sketch the graph of $(x - y)(x + y) = 0$ and explain how it is related to the graphs of $x - y = 0$ and $x + y = 0$.

54. Graph $F = \frac{9}{5}C + 32$ in a CF-coordinate system.

55. Graph $u = 3v^2$ in a uv-coordinate system.

56. Graph $Y = 4X + 5$ in a YX-coordinate system.

57. A point moves in the xy-plane in such a way that at any time t its coordinates are given by $x = 5t + 2$ and $y = t - 3$. By expressing y in terms of x, show that the point moves along a straight line.

58. A point moves in the xy-plane in such a way that at any time t its coordinates are given by $x = 1 + 3t^2$ and $y = 2 - t^2$. By expressing y in terms of x, show that the point moves along a straight-line path and specify the values of x for which the equation is valid.

59. Find the area of the triangle formed by the coordinate axes and the line through $(1, 4)$ and $(2, 1)$.

60. Draw the graph of $4x^2 - 9y^2 = 0$.

61. In each part, name an appropriate coordinate system for graphing the equation [e.g., an $\alpha\beta$-coordinate system in part (a)], and state whether the graph of the equation is a line in that coordinate system.
(a) $3\alpha - 2\beta = 5$
(b) $A = 2000(1 + 0.06t)$
(c) $A = \pi r^2$
(d) $E = mc^2$ (c constant)
(e) $V = C(1 - rt)$ (r and C constant)
(f) $V = \frac{1}{3}\pi r^2 h$ (r constant)
(g) $V = \frac{1}{3}\pi r^2 h$ (h constant)

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