MATH 251 Winter 2005

Schedule for the review by E. Kudryavtseva:

Thursday, April 21, 11:00–13:00, ST 147.

REVIEW QUESTIONS FOR FINAL EXAMINATION

1. Differentiate: (a) $(\cos^2 x + \sin^2 x)^{300}$, (b) $\arccos(5x)$, (c) $\arcsin\sqrt{1-x^2}$,

   (d) $xe^x \sin x$, (e) $x\ln(x+1)$, (f) $x^{(x^2)}$, (g) $\dfrac{d}{dx}\displaystyle\int_4^5 e^{-t^2}dt$, (h) $\dfrac{d}{dx}\displaystyle\int_4^x e^{-t^2}dt$.

2. Find the integrals (a) $\int(\cos^2 x + \sin^2 x)^{300}dx$, (b) $\int(2x + x^{-3} + e^{2x})\, dx$,

   (c) $\displaystyle\int \frac{(\ln x)^7}{x}dx$, (d) $\displaystyle\int \cos(3x)dx$, (e) $\displaystyle\int \frac{x^4+1}{x^3}dx$, (f) $\displaystyle\int_0^{\pi/4} \frac{dx}{\cos^2 x}$.

3. Evaluate the following limits:

   (a) $\displaystyle\lim_{x\to 2}\frac{\sqrt{x^2+1}-\sqrt{5}}{x-2}$, (b) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{x^2+x+1}}{x}$, (c) $\displaystyle\lim_{x\to-\infty}\frac{x^2+\sin x}{2x^2+1}$,

   (d) $\displaystyle\lim_{x\to 0}\frac{e^x-1-x}{x^2}$, (e) $\displaystyle\lim_{x\to\pi}\frac{2\sin(\frac{x}{6})-1}{x-\pi}$

4. Find the domain and range of (a) $f(x)=\sqrt{x^2-9}$, (b) $g(x)=\ln(x^2-5x+4)$.

5. The position of a moving body is given as a function of the time t by $s(t)=t^3-4t^2+t$ for $t>0$.

   (a) Find the velocity, $v(t)$, and the acceleration, $a(t)$, of the body.

   (b) Find the average velocity, v_{av}, of the body between the times $t=1$ and $t=2$.

   (c) Find a time $t_0 \in (1,2)$ when the velocity $v(t_0)$ equals the average velocity v_{av} found in part (b).

   (d) Which theorem guarantees the existence of the t_0 in (c)?

   (e) Find all time intervals for $t>0$ when the body is slowing down.

6. (a) Find the linearization of $f(x)=\sqrt[5]{31+x}$ around $x_0=1$.

   (b) Use it to estimate $\sqrt[5]{30}$.

   (c) Is the estimate found in part (b) an overestimate or underestimate of the exact value of $\sqrt[5]{30}$? Give reasons for your answer.

7. A right triangle has hypotenuse 5. Find the dimensions that maximize its perimeter.

8. Derive the differentiation formula for $y=\arcsin x$.

9. Find the tangent line to the curve $x^5+y^5+y\cos x+6x=2$ at $P(0,1)$.

10. Find the area under the curve $y = \frac{1}{3x \ln x}$, $e^2 \le x \le e^3$.

11. A man who is $2m$ tall stands $6m$ away from the base of a tall pole. A lamp is moving up the pole at the steady rate $.5\ m/sec$. Find how fast the length of the man's shadow is changing at the moment the lamp is $4m$ above the ground.

12. Sketch the graph of the function $y = f(x) = \frac{x+2}{x^2} - 1$.

    (a) Find the domain, range, asymptotes.

    (b) Show that $f'(x) = -\frac{x+4}{x^3}$, $f''(x) = \frac{2x+12}{x^4}$.

    (c) Find the intervals where f is increasing or decreasing.

    (d) Find the intervals where f is concave up or concave down.

    (e) Find the (x, y)-coordinates of all local minima, local maxima, inflexion points, x- and y-intercepts, and indicate them on the graph.

13. Determine a and b so that $f(x) = ax^3 + bx^2 + 3x - 17$ has the point $(1, -18)$ as an inflexion point.

ANSWERS

1. (a) 0, (b) $-\frac{5}{\sqrt{1-25x^2}}$, (c) $-\frac{\operatorname{sgn} x}{\sqrt{1-x^2}}$, (d) $e^x(\sin x+x\sin x+x\cos x)$, (e) $\ln(x+1)+\frac{x}{x+1}$,
   (f) $2x^{(x^2+1)}\ln x$, (g) 0, (h) e^{-x^2}.

2. (a) $x+C$, (b) $x^2-\frac{1}{2}x^{-2}+\frac{1}{2}e^{2x}+C$, (c) $\frac{1}{8}(\ln x)^8+C$, (d) $\frac{1}{3}\sin(3x)+C$,
   (e) $\frac{x^2}{2}-\frac{1}{2x^2}+C$, (f) 1.

3. (a) $\frac{2}{\sqrt{5}}$, (b) -1, (c) $\frac{1}{2}$, (d) $\frac{1}{2}$, (e) $\frac{1}{2\sqrt{3}}$.

4. (a) $D_f=(-\infty,-3]\cup[3,\infty)$, $R_f=[0,\infty)$,
   (b) $D_g=(-\infty,1)\cup(4,\infty)$, $R_g=(-\infty,\infty)=\mathbb{R}$.

5. (a) $v(t)=3t^2-8t+1$, $a(t)=6t-8$, (b) $v_{\text{av}}=-4$, (c) $t_0=\frac{5}{3}$,
   (d) Mean Value Theorem (MVT), (e) $(0,\frac{4}{3})$.

6. (a) $L(x)=2+\frac{(x-1)}{80}=\frac{x+159}{80}$, (b) $\sqrt[5]{30}\approx\frac{79}{40}$,
   (c) overestimate, since (Error)$=\sqrt[5]{30}-\frac{79}{40}$ is negative (why?).

7. $\sqrt{5/2}$ and $\sqrt{5/2}$.

8. METHOD 1: $y(x)=\arcsin x$, $\sin(y(x))=x$. Now by the Chain Rule, $\cos(y(x))\cdot y'(x)=1$, so $y'(x)=\frac{1}{\cos(y(x))}=\frac{1}{\cos(\arcsin x)}$.

   Let us **simplify** $\cos(\arcsin x)$. Using the **Right triangle trick** (draw the picture!), we have $\cos(\arcsin x)=\sqrt{1-x^2}$, so $y'(x)=\frac{1}{\sqrt{1-x^2}}$.

   METHOD 2: Denote $y=g(x)=\arcsin x$, $x=f(y)=\sin y$, so the functions f and g (defined on appropriate intervals) are inverses of each other. Now $f'(y)=\cos y$. By the derivative formula for the inverse function, we have

   $$g'(x)=\left.\frac{1}{f'(y)}\right|_{y=g(x)}=\frac{1}{f'(g(x))},$$

   thus $g'(x)=\frac{1}{\cos(\arcsin x)}$. Now we **simplify** $\cos(\arcsin x)$ as above.

9. $y=1-x$.

10. $\frac{1}{3}\ln(\frac{3}{2})$.

11. $-1.5\ m/sec$.

12. (a) $D_f=(-\infty,0)\cup(0,\infty)$, $R_f=[-\frac{9}{8},\infty)$,
    vertical asymptote $x=0$, horizontal asymptote $y=-1$,
    (c) increasing on $(-4,0)$, decreasing on $(-\infty,-4)$ and $(0,\infty)$,
    (d) concave up on $(-6,0)$ and $(0,\infty)$, concave down on $(-\infty,-6)$,
    (e) local minimum $(-4,-\frac{9}{8})$, no local maxima, inflexion point $(-6,-\frac{10}{9})$,
    x-intercepts $(-1,0)$ and $(2,0)$, no y-intercepts.

13. $a=2$, $b=-6$.