REVIEW FINAL EXAMINATION MATH 251

1. Differentiate: $\arccos(5x)$, $\arcsin\sqrt{1-x^2}$, $xe^x\sin x$, $x\ln x$, $(\cos x)^x$

2. $\lim\limits_{x\to 3}\dfrac{\sqrt{x^2+2}-\sqrt{11}}{x-3}$, $\lim\limits_{x\to 1}\dfrac{\ln x}{x^2-5x+4}$, $\lim\limits_{x\to 0}\dfrac{2\sin x-3\cos x}{2\cos x-3\sin x}$

3. $\int x(x^2-1)^7dx$, $\int\cos(3x)\,dx$, $\int\dfrac{x^4+1}{x^3}\,dx$

4. T-F $e^{2\ln 5}=25$ ___ ; $5^{2\ln e}=25$ ___ ; $f'(a)=0$, $f''(a)>0$ implies a rel min at $x=a$ ___ ; $f''(a)=0$ implies an inflexion pt at $x=a$ ___ ; $\arcsin(\sin(7\pi/4))=7\pi/4$ ___ ; If $g(x)\geq f(x)$ then $g'(x)\geq f'(x)$ ___ ; $\sqrt{x^2}=x$ ___ ; $g(x)\geq f(x)$ implies $\int_a^b g(x)\,dx\geq\int_a^b f(x)\,dx$ ___ ; $\cos^2 x\leq|\cos x|$ ___ ; $\int_{-e}^{-1}dx/x=-1$ ___ .

5. Use the method of linear approximation (differentials) to estimate $126^{1/3}$.

6. Jane and John each work out a certain indefinite integral. Jane's answer is $(-x+3)/(x^2+x-2)$, while John's is $(x^2+1)/(x^2+x-2)$. Both answers are correct. Explain how this is possible.

7. Find the tangent line to the curve $x^5+y^5+y\cos x+6x=2$ at $P=(0,1)$.

8. Find the area under the curve $y=xe^{x^2}$, $0\leq x\leq\sqrt{\ln(34)}$

9. A man who is 2m tall stands 6m away from the base of a tall pole. A lamp is moving up the pole at the steady rate .5 m/sec. Find how fast the length of the man's shadow is changing at the moment the lamp is 4m above the ground.

10. Determine the point on the parabola $y=x^2$ closest to the point $Q=(18,0)$.

Solutions 1. $\dfrac{-5}{\sqrt{1-25x^2}}$, $\dfrac{-1}{\sqrt{1-x^2}}$, $e^x(\sin x+x\sin x+x\cos x)$, $\ln x+1$, $(\cos x)^x\begin{pmatrix}\ln(\cos x)-x\tan x\\ \cancel{\cos x-x\sin x}\end{pmatrix}$

2. $3/\sqrt{11}$, $-1/3$, $-3/2$ 3. $\frac{1}{16}(x^2-1)^8+C$, $\frac{1}{3}\sin(3x)+C$ $\frac{1}{2}x^2-\frac{1}{2x^2}+C$

4. T; T, T, F, F, F, F, T, T, T 5. 376/75

6. Take the difference of the two answers, and observe it equals a constant.

7. $y=-x+1$ 8. $33/2$ 9. $-3/2$ m/sec 10. $(2,4)$