MATH 251

PRACTICE MIDTERM

1. Solve the following inequality:

$$\text{(a) } |x-2|<1, \qquad \text{(b) } x-2 \le \frac{-1}{x+1}.$$

2. Find all horizontal and vertical asymptotes of

$$\text{(a) } f(x)=\frac{x^2}{x-1}-\frac{x^2+1}{x+1}, \qquad \text{(b) } f(x)=\frac{\sqrt{9x^2+x+5}}{x+2}.$$

3. Find the following limits:

$$\text{(a) } \lim_{x\to-\infty}\frac{x^8-3x^5+2}{2x^8+x^5-3}, \qquad \text{(b) } \lim_{x\to2}\frac{x^3-x^2-4}{x^2-4}, \qquad \text{(c) } \lim_{x\to-2^-}\frac{2x^2+3x+1}{x^3+x^2+4},$$

$$\text{(d) } \lim_{x\to0}\frac{|x-3|-3}{2x}, \qquad \text{(e) } \lim_{x\to\infty}\frac{3x-\cos x}{x}, \qquad \text{(f) } \lim_{x\to\frac{\pi}{4}}\frac{\tan x-\tan\frac{\pi}{4}}{x-\frac{\pi}{4}},$$

$$\text{(g) } \lim_{x\to\infty} x\sin\frac{1}{x}, \qquad \text{(h) } \lim_{x\to0} x\sin\frac{1}{x}, \qquad \text{(i) } \lim_{x\to0}\sin\frac{1}{x}.$$

4. Determine $k\in\mathbb{R}$ such that the function $g(x)=\begin{cases} x^2-4k & \text{if } x\le1, \\ x^3+k^2+4 & \text{if } x>1 \end{cases}$ is continuous.

5. Differentiate the following functions. If possible, simplify your answers.

$$\text{(a) } y=3x^4-10x^{6/5}, \qquad \text{(b) } y=\frac{-2}{x^3+2}, \qquad \text{(c) } y=\frac{x^2+1}{x^3-x+5},$$

$$\text{(d) } y=|2x|, \qquad \text{(e) } y=x^2\sin(3x)+\cot^2(\frac{1}{x}), \qquad \text{(f) } y=\frac{\tan(3x)}{x+7},$$

$$\text{(g) } y=\frac{\cos x+1}{\sin x+1}, \qquad \text{(h) } y=\cos^3(x)\cos(x^3), \qquad \text{(i) } y=\sec^2(x^2+5x),$$

$$\text{(j) } y=\csc(\sqrt{x^2+x}), \qquad \text{(k) } y=\sin^3(\cos x).$$

6. (a) Estimate $\sqrt{103}$ using differentials.
(b) Find the linear approximation to $f(x)=-x^2+\cos x$ at $x_0=\pi$. Find the Error of this linear approximation at $x=-\pi$.

7. Find all intervals where the function $f(x)=x^3-3x^2-6x+2$ is increasing or decreasing.

8. Given the data in the following table, determine $(g \circ f)'(2)$

x	-1	0	2	5
$f(x)$	2	1	-1	7
$g(x)$	5	2	0	4
$f'(x)$	0	2	3	-1
$g'(x)$	4	5	2	5

9. Find the equation of the tangent line to the graph of

(a) $\tan(xy) + \sin y = x^2 y + 2x - 2$ at the point $(1, 0)$;

(b) $y^2 + \sin y = x$ at the point (π^2, π).

10. Answer TRUE or FALSE for questions (a)–(m). **Do not write "T, F".**

(a) If x_1, x_2 are any non-zero numbers and $x_1 < x_2$ then it follows that $\frac{1}{x_1} > \frac{1}{x_2}$. ________________

(b) $2.\overline{9} < 3$ ________________

(c) If f, g are discontinuous at $x = a$ then $f + g$ is discontinuous at $x = a$. ________________

(d) If $\lim_{x \to a} f(x) = \infty$ then f can not be continuous at $x = a$. ________________

(e) If $g(x) = (f(x))^2$ is continuous then $f(x)$ must also be continuous. ________________

(f) The Intermediate-Value Theorem allows us to conclude that the equation $3x^3 - x + 4 = 5$ has a solution in the interval
a: $[-1, 0]$, b: $[0, 1]$, c: $[1, 2]$, d: None of these. ________________

(g) If f is continuous then f is differentiable. ________________

(h) If $f(x) > g(x)$ then $f'(x) > g'(x)$. ________________

(i) $(fgh)' = f'gh + fg'h + fgh'$. ________________

(j) The function $f(x) = \sin x$ is continuous for all x. ________________

(k) If $\frac{\pi}{4} < x < \frac{\pi}{2}$ then $\tan x > 1$. ________________

(l) $\tan(\frac{21\pi}{4}) > 0$ ________________

(m) $\tan(3.24) > 0$ ________________

(n) If $f(x) = \frac{2x-3}{x-2}$ for $x \neq 2$ then the range of f is ________________

(o) If $f(x) = \frac{1}{x+2}$ for $x \neq -2$ and $(f \circ g)(x) = x$ then $g(x) =$ ________________