MATH 251 B47/B48/B50 Quiz #1 Winter 2005

Name: K E Y I.D.#: _____

Answer all questions. Calculators are NOT allowed. 30 minutes.

1. Answer TRUE or FALSE for each of the following questions.
WRITE "True" OR "False", DO NOT USE "T,F". [50]

(a) $|x+3| \geq |x|$. FALSE

(b) $\lfloor -2.154 \rfloor = -2$. FALSE

(c) $g(x) = 2x\sin(\frac{3\pi}{2} - x)$ is an odd function. TRUE

(d) $|a-b| < |a| + |b|$. FALSE

(e) $1.\overline{9} < 2$. FALSE

(f) The solution set of the inequality $25x^2 - 10x + 1 \geq 0$ consists of all real numbers x. TRUE

(g) The solution set of the inequality $25x^2 - 10x + 1 \leq 0$ is the empty set $\emptyset$ (i.e. there are no solutions). FALSE

(h) The domain of the function $f(x) = \frac{\sqrt{x-5}}{x(x-8)}$ is $[5,8) \cup (8,\infty)$. TRUE

(i) The solution set of $|7x^4 - 2x^3 + 13x + 4| < -2|x+1|$ is the empty set $\emptyset$. TRUE

(j) If P is a point in the third quadrant, and Q is a point in the second quadrant, then the line PQ joining them has positive slope. FALSE

TURN OVER FOR QUESTIONS 2 AND 3

For questions 2 and 3, show your work in the space provided.

2. Solve the inequality $\frac{x+1}{2} \geq \frac{3}{x+2}$. [25]

Solution: First bring everything to one side:

$$\frac{x+1}{2} - \frac{3}{x+2} \geq 0.$$

Simplify:

$$\frac{(x+1)(x+2)-6}{2(x+2)} \geq 0; \qquad \frac{x^2+3x-4}{2(x+2)} \geq 0; \qquad \frac{(x-1)(x+4)}{2(x+2)} \geq 0.$$

Testing:

$$\underline{\quad - \quad}_{-4}\underline{\quad + \quad}_{-2}\underline{\quad - \quad}_{1}\underline{\quad + \quad}$$

The point x=-2 does not belong to the solution set, since it is not in the domain.

Answer: $x \in [-4,-2) \cup (1,\infty)$.

3. Find an equation of the line parallel to $3x-2y=1$ passing through the centre of the circle $x^2-4x+y^2+6y=3$. [25]

Solution: Complete the squares:

$$(x-2)^2-4+(y+3)^2-9=3.$$

Thus

$$(x-2)^2+(y+3)^2=16.$$

So, the point $(2,-3)$ is the centre of the circle. We can now proceed in two ways.

Method 1: Any line parallel to $3x-2y=1$ has an equation with $3x-2y$ on the left hand side, and some constant on the right hand side. To find this constant, let us substitute the coordinates $(2,-3)$. We get $3x-2y=3\cdot 2-2(-3)$, so $3x-2y=12$.

Method 2: The line $3x-2y=1$ has slope $m=\frac{3}{2}$, so any line parallel to it has the same slope. Now, an equation of the line going through the point $(2,-3)$ with the slope $m=\frac{3}{2}$ is

$$y=-3+m(x-2), \qquad y=-3+\frac{3}{2}(x-2), \qquad y=\frac{3}{2}x-6.$$

Answer: $3x-2y=12$ OR $y=\frac{3}{2}x-6$.

END OF PAPER