MATH 251 B47/B48/B50 Quiz #5 Winter 2005

Name:___________**K E Y**___________ **I.D.#:**__________________

Answer all questions. Calculators are NOT allowed. 30 minutes.

1. Calculate the Taylor polynomial of degree 4 for $f(x) = \frac{1}{x-1}$ about $x = 0$. [15]

Solution: We need 4 derivatives of f at $x = 0$:

$$f'(x) = -\frac{1}{(x-1)^2}, \qquad f''(x) = \frac{2}{(x-1)^3}, \qquad f'''(x) = -\frac{6}{(x-1)^4}, \qquad f''''(x) = \frac{24}{(x-1)^5},$$

now evaluate at $x = 0$:

$$f(0) = -1, \qquad f'(0) = -1, \qquad f''(0) = -2, \qquad f'''(0) = -6, \qquad f''''(0) = -24,$$

so

$$P_4(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3 + \frac{f''''(0)}{4!}(x-0)^4$$

$$= -1 - x + \frac{-2}{2!}x^2 + \frac{-6}{3!}x^3 + \frac{-24}{4!}x^4 = -1 - x - x^2 - x^3 - x^4.$$

Answer: $P_4(x) = -1 - x - x^2 - x^3 - x^4$.

2. Evaluate [20]

$$\lim_{x\to 0} \frac{\cos x - 1}{x^2} = \text{“}\frac{1-1}{0}\text{”} = \text{“}\frac{0}{0}\text{”} = \text{(by l'Hôpital's Rule)} = \lim_{x\to 0} \frac{(\cos x - 1)'}{(x^2)'}$$

$$= \lim_{x\to 0} \frac{-\sin x}{2x} = \text{“}\frac{0}{0}\text{”} = \text{(by l'Hôpital's Rule)} = \lim_{x\to 0} \frac{-\cos x}{2} = \frac{-1}{2} = -\frac{1}{2}.$$

Answer: $-\frac{1}{2}$.

3. Write in sigma notation [15]

$$1 - \cos(x) + \cos(2x) - \cos(3x) + \ldots - \cos(11x) + \cos(12x) = \sum_{j=0}^{12} (-1)^j \cos(jx).$$

TURN OVER FOR QUESTIONS 4 AND 5

4. How fast will the string be let out on a kite that is flying $50m$ high and $120m$ distance horizontally from the kite flyer, if the kite is moving $10m/sec$ in the horizontal direction, away from the kite flyer? [30]

Solution: Denote the sides of our right triangle by x (the horizontal side), y (the vertical side), and z (the hypothenus). By the **Pythagorean Theorem**, we have the **basic relation**:

$$x^2 + y^2 = z^2.$$

Let us differentiate this basic relation with respect to time t using the Chain Rule, remembering that x, y, and z are functions of t:

$$2xx' + 2yy' = 2zz', \qquad xx' + yy' = zz', \qquad z' = \frac{xx' + yy'}{z}.$$

We want to find z' at a certain time t_0 when $x = 120$ and $y = 50$, so $z = \sqrt{120^2 + 50^2} = \sqrt{14400 + 2500} = \sqrt{16900} = 130$. We also know that, at time $t = t_0$, $x' = 10$ and $y' = 0$ (since the kite is moving in the horizontal direction, so its vertical velocity y' is zero).

Now evaluate at $t = t_0$:

$$z'(t_0) = \left.\frac{xx' + yy'}{z}\right|_{t=t_0} = \frac{120 \cdot 10 + 50 \cdot 0}{130} = \frac{1200}{130} = \frac{120}{13}.$$

Answer: $\frac{120}{13}$ $m/sec.$

5. Evaluate [20]

$$\sum_{j=1}^{n}(3^j + 2) = \left(\sum_{j=1}^{n} 3^j\right) + \left(\sum_{j=1}^{n} 2\right)$$

$$= (3 + 3^2 + 3^3 + \ldots + 3^{n-1} + 3^n) + \underbrace{(2 + 2 + \ldots + 2 + 2)}_{\longleftarrow\ n \text{ times}\ \longrightarrow}$$

$$= \frac{3^{n+1} - 3}{3 - 1} + 2n = \frac{3^{n+1} - 3}{2} + 2n.$$

END OF PAPER