MATH 251 B47/B48/B50 Quiz #a Winter 2005

Name: K E Y I.D.#:

Answer all questions. Calculators are NOT allowed. 30 minutes.

1. Differentiate the following functions: [30]

   (a) $\dfrac{4}{\sqrt{x}}+5\sqrt{x}-\dfrac{1}{\sqrt[5]{x^3}}$ $\qquad -\dfrac{2}{\sqrt{x}^3}+\dfrac{5}{2\sqrt{x}}+\dfrac{3}{5\sqrt[5]{x^8}}$

   (b) $\dfrac{3x+5}{4x-7}$ $\qquad \dfrac{3(4x-7)-4(3x+5)}{(4x-7)^2}=\dfrac{-21-20}{(4x-7)^2}=\dfrac{-41}{(4x-7)^2}$

   (c) $\sqrt{x^3+3x+1}$ $\qquad \dfrac{3x^2+3}{2\sqrt{x^3+3x+1}}$

   (d) $\cos(4x^2+3x)$ $\qquad -\sin(4x^2+3x)\cdot(8x+3)=-(8x+3)\sin(4x^2+3x)$

2. For $f(x)=\dfrac{x}{x+2}$, $g(x)=\dfrac{x+1}{x-1}$ find the composition $f\circ g$ and its domain $D_{f\circ g}$. [20]

   Solution:

$$f\circ g(x)=\frac{g(x)}{g(x)+2}=\frac{\frac{x+1}{x-1}}{\frac{x+1}{x-1}+2}$$

   and this function is defined for all $x\neq 1$. We simplify this function for $x\neq 1$:

$$=\frac{x+1}{x+1+2x-2}=\frac{x+1}{3x-1}.$$

   Thus the domain is $D_{f\circ g}=(-\infty,\frac{1}{3})\cup(\frac{1}{3},1)\cup(1,\infty)$.

TURN OVER FOR QUESTIONS 3, 4, AND 5

3. The limit $\lim_{h\to 0} \dfrac{\sqrt{9+3h}-3}{h}$ represents the derivative of a certain function f at a certain point a. Which of the following is correct? [15]

(a) $f(x) = \sqrt{x},\ a = 3,$

(b) $f(x) = \sqrt{x},\ a = 9,$

(c) $f(x) = 3\sqrt{x},\ a = 9,$

(d) $f(x) = \sqrt{3x},\ a = 3,$ then $f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\sqrt{9+3h}-3}{h}$.

(e) None of the above.

4. Let $f(x) = x + \frac{4}{x^2}$. Find a value of x in $(1, 2)$ such that the tangent line to the graph of f at $(x, f(x))$ is parallel to the secant line which cuts the curve at $x = 1$ and $x = 2$. [20]

Solution: Parallel means having the same slope. The slope of the secant line at $x = 1$ and $x = 2$ equals

$$m = \frac{f(2)-f(1)}{2-1} = \frac{2+\frac{4}{4}-(1+4)}{1} = 2+1-1-4 = -2.$$

The slope of the tangent line at the point $(x, f(x))$ equals

$$f'(x) = 1 - \frac{4\cdot 2}{x^3} = 1 - \frac{8}{x^3}.$$

So we must find $x \in (1, 2)$ such that $f'(x) = m$, so $1 - \frac{8}{x^3} = -2$ (such x exists by the Mean value Theorem). Let us solve this equation for x:

$$1 - \frac{8}{x^3} = -2, \qquad \frac{8}{x^3} = 3, \qquad x^3 = \frac{8}{3}, \qquad x = \sqrt[3]{\frac{8}{3}} = \frac{\sqrt[3]{8}}{\sqrt[3]{3}} = \frac{2}{\sqrt[3]{3}}.$$

Let us check that $\frac{2}{\sqrt[3]{3}} \in (1, 2)$: this follows from $1 < \frac{8}{3} < 8$.

Answer: $x = \frac{2}{\sqrt[3]{3}}$.

5. Find all intervals where the function $f(x) = x^3 + 3x^2 - 9x + 1$ is increasing or decreasing. [15]

Solution: By a consequence of the Mean Value Theorem, f is increasing if $f'(x) > 0$, and decreasing if $f'(x) < 0$. Now

$$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x-1)(x+3).$$

So, the splitting points for $f'(x)$ are $x = 1$ and $x = -3$. Testing:

		-3		1	
f' :	$+$		$-$		$+$

So,

f :	increases	decreases	increases

Answer: f is increasing on $(-\infty, -3]$ and $[1, \infty)$, and decreasing on $[-3, 1]$.

END OF PAPER