MATH 251 B45/B46 Quiz #a Winter 2005

Name: K E Y I.D.#:

Answer all questions. Calculators are NOT allowed. 30 minutes.

1. Differentiate the following functions: [30]

   (a) $x^3+2x^{\frac{5}{4}}-\frac{1}{x^3}-3$ $\qquad 3x^2+\frac{5}{2}x^{\frac{1}{4}}+\frac{3}{x^4}$

   (b) $\frac{x}{x^2+1}$ $\qquad \frac{x^2+1-2x^2}{(x^2+1)^2}=\frac{1-x^2}{(1+x^2)^2}$

   (c) $(x^3+2x-1)^{30}$ $\qquad 30(x^3+2x-1)^{29}(3x^2+2)$

   (d) $(2x+1)\sec x$ $\qquad 2\sec x+(2x+1)\sec x\tan x$

2. For $f(x)=\frac{x}{x+2}$, $g(x)=\frac{x+1}{x-1}$ find the composition $g\circ f$ and its domain $D_{g\circ f}$. [20]

   Solution:

$$g\circ f(x)=\frac{f(x)+1}{f(x)-1}=\frac{\frac{x}{x+2}+1}{\frac{x}{x+2}-1}$$

   and this function is defined for all $x\neq -2$. We simplify this function for $x\neq -2$:

$$=\frac{x+x+2}{x-x-2}=\frac{2x+2}{-2}=-x-1.$$

   Thus the domain is $D_{g\circ f}=(-\infty,-2)\cup(-2,\infty)$.

TURN OVER FOR QUESTIONS 3, 4, AND 5

3. The limit $\lim_{h\to 0} \frac{\sqrt{4+2h}-2}{h}$ represents the derivative of a certain function f at a certain point a. Which of the following is correct? [15]

(a) $f(x) = \sqrt{x},\ a = 2,$

(b) $f(x) = \sqrt{x},\ a = 4,$

(c) $f(x) = 2\sqrt{x},\ a = 4,$

(d) $f(x) = \sqrt{2x},\ a = 2,$ then $f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\sqrt{4+2h}-2}{h}$.

(e) None of the above.

4. The displacement of a particle at time t is given by $s(t) = t^3 - t$. [20]

(a) Find the average velocity over the time interval $1 \le t \le 3$.

Solution: $\frac{s(3)-s(1)}{3-1} = \frac{3^3-3-(1-1)}{2} = \frac{27-3}{2} = \frac{24}{2} = 12.$

Answer: 12.

(b) For which value of $t \in (1,3)$ will the instantaneous velocity equal this average velocity?

Solution: We have to find $t \in (1,3)$ such that the instantaneous velocity $v(t)$ equals 12 (such t exists due to the Mean Value Theorem). Since $v(t) = s'(t) = 3t^2 - 1$, we have to find $t \in (1,3)$ such that $3t^2 - 1 = 12$, so $3t^2 = 13$, so $t = \pm\sqrt{\frac{13}{3}}$. Thus for both values $t = \sqrt{\frac{13}{3}}$ and $t = -\sqrt{\frac{13}{3}}$ the instantaneous velocity equals 12. Now $-\sqrt{\frac{13}{3}}$ does not belong to the interval $(1,3)$, but $\sqrt{\frac{13}{3}} \in (1,3)$, since $1 < \frac{13}{3} < 9$.

Answer: $t = \sqrt{\frac{13}{3}}$.

5. Find all intervals where the function $f(x) = 2x^3 + 3x^2 - 36x + 4$ is increasing or decreasing. [15]

Solution: By a consequence of the Mean Value Theorem, f is increasing if $f'(x) > 0$, and decreasing if $f'(x) < 0$. Now

$$f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6) = 6(x-2)(x+3).$$

So, the splitting points for $f'(x)$ are $x = 2$ and $x = -3$. Testing:

		-3		2	
f' :	$+$		$-$		$+$

So,

f :	increases	decreases	increases

Answer: f is increasing on $(-\infty, -3]$ and $[2, \infty)$, and decreasing on $[-3, 2]$.

END OF PAPER