MATH 251

WORK SHEET #1 - SOLUTIONS

1. Solve the inequality

a: $|3x+1| < |x-2|$, b: $\frac{2}{x+3} \geq \frac{1}{2}$.

a. Since $|\ldots|$ is always positive or zero, we can square both sides and the sign of the inequality stays the same: $(3x+1)^2 < (x-2)^2$, since $|\ldots|^2 = (\ldots)^2$. Now $9x^2+6x+1 < x^2-4x+4$, **everithing on one side**: $8x^2+10x-3 < 0$; $(4x-1)(2x+3) < 0$. Thus **split points** are: $x = -\frac{3}{2}, \frac{1}{4}$.
Testing sign at intermediate points:

$$\underline{\quad + \quad}_{-\frac{3}{2}}\underline{\quad - \quad}_{\frac{1}{4}}\underline{\quad + \quad}$$

The solution: $x \in]-\frac{3}{2}, \frac{1}{4}[$.

b. **Everything on one side** and common denominator: $\frac{4-x-3}{(x+3)2} \geq 0$, **simplify**: $\frac{1-x}{x+3} \geq 0$. So, **split points** are: $x = -3, 1$. **Testing**:

$$\underline{\quad - \quad}_{-\hat{3}}\underline{\quad + \quad}_{1}\underline{\quad - \quad}$$

The solution: $x \in]-3, 1]$.

2. Find the radius and the centre of the circle $x^2 - 4x + y^2 + 2y = 11$.

Complete the squares: $(x-2)^2-4+(y+1)^2-1 = 11$, so $(x-2)^2+(y+1)^2 = 16$, thus $r = 4$ and the point $(2,-1)$ is the centre.

3. Solve the inequality

a: $|x-1|+2 > 0$, b: $\frac{3}{x-1} \leq \frac{2}{x-3}$.

a. Since $|\ldots|$ is always positive or zero, $|\ldots|+2$ is always positive, so the solution is $x \in]-\infty, +\infty[= \mathbb{R}$.

b. **Everything on one side** and common denominator: $\frac{3(x-3)-2(x-1)}{(x-1)(x-3)} \leq 0$, **simplify**: $\frac{x-7}{(x-1)(x-3)} \leq 0$. So, **split points** are: $x = 1, 3, 7$. **Testing**:

$$\underline{\quad - \quad}_{\hat{1}}\underline{\quad + \quad}_{\hat{3}}\underline{\quad - \quad}_{7}\underline{\quad + \quad}$$

The solution: $x \in]-\infty, 1[\cup]3, 7]$.

4. Given four lines $l_1 : 3x-2y = 1$, $l_2 : 2y+3x = 0$, $l_3 : 3x+2y = 3$, $l_4 : 2x+3y = 2$. Choose all which are

a: parallel, b: perpendicular.

Find slopes: $m_1 = \frac{3}{2}$, $m_2 = -\frac{3}{2}$, $m_3 = -\frac{3}{2}$, $m_1 = -\frac{2}{3}$. So,
a: $l_2 \| l_3$, since they have the same slope, and b: $l_1 \perp l_4$, since $m_1 \cdot m_4 = -1$.

5. Solve the inequality
a: $\frac{1}{x+1} \geq 1 + x$, b: $|3x+2| > 0$.

b. Since $| \ldots |$ is always positive or zero, we have to eliminate zero: $3x+2=0$ for $x = -\frac{2}{3}$. The solutions: $x \neq -\frac{2}{3}$ or $x \in]-\infty, -\frac{2}{3}[\cup]-\frac{2}{3}, +\infty[$. (It is better to use the latter way for writing the answer.)

a. **Everything on one side** and common denominator: $\frac{1-x^2-2x-1}{x+1} \geq 0$, **simplify**: $\frac{-x^2-2x}{x+1} \geq 0$; $\frac{-x(x+2)}{x+1} \geq 0$. So, **split points** are: $x = -1, 0, -2$. **Testing**:

$$\underline{\quad + \quad}\,{\scriptstyle -2}\,\underline{\quad - \quad}\,{\scriptstyle -\hat{1}}\,\underline{\quad + \quad}\,{\scriptstyle 0}\,\underline{\quad - \quad}$$

The solution: $x \in]-\infty, -2] \cup]-1, 0]$.

6. Find an equation of the line perpendicular to the x-axis passing through the point $(1, -3)$.

$\perp$ to the x-axis means a vertical line, so $x = 1$. (What is y? y is any.)

7. Solve the inequality
a: $2x + 7 > x^2$, b: $\frac{x}{2} < \frac{3}{x+2}$.

a. **Everything on one side**: $0 > x^2 - 2x - 7$. Now find the roots, first discriminant $D = 4 + 4 \cdot 7 = 32 > 0$, so using the formula, the roots are $x_1 = \frac{2-\sqrt{32}}{2} = 1 - 2\sqrt{2}$ and $x_2 = \frac{2+\sqrt{32}}{2} = 1 + 2\sqrt{2}$. Now **testing**:

$$\underline{\quad + \quad}\,{\scriptstyle x_1}\,\underline{\quad - \quad}\,{\scriptstyle x_2}\,\underline{\quad + \quad}$$

OR parabola open up is below the x-axis if $x \in]x_1, x_2[=]1 - 2\sqrt{2}, 1 + 2\sqrt{2}[$.

b. **Everything on one side** and common denominator: $\frac{x(x+2)-6}{2(x+2)} < 0$, **simplify**: $\frac{x^2+2x-6}{2(x+2)} < 0$. Roots are $x_1 = -1 - \sqrt{1+6} = -1 - \sqrt{7}$ and $x_2 = 1 + \sqrt{7}$. So, **split points** are: $x = -1 - \sqrt{7}, -2, -1 + \sqrt{7}$. **Testing**:

$$\underline{\quad - \quad}\,{\scriptstyle -1-\sqrt{7}}\,\underline{\quad + \quad}\,{\scriptstyle -\hat{2}}\,\underline{\quad - \quad}\,{\scriptstyle -1+\sqrt{7}}\,\underline{\quad + \quad}$$

The solution: $x \in]-\infty, -1-\sqrt{7}[\cup]-2, -1+\sqrt{7}[$.

8. Find the vertex of the parabola $y + 2x^2 + 6x + 1 = 0$.

Complete the square: $y + 2(x + \frac{3}{2})^2 - \frac{9}{2} + 1 = 0$; thus $y = -2(x + \frac{3}{2})^2 + \frac{7}{2}$. So, the point $(-\frac{3}{2}, \frac{7}{2})$ is the vertex of the parabola. (Sketch the graph.)

9. Answer TRUE or FALSE for each of the following questions:

a: $\lfloor -3.107 \rfloor = -3$, b: $0.\overline{215} < \frac{215}{999}$,

c: $f(x) = x\cos(\frac{3\pi}{2} + x)$ is an even function.

a: $\lfloor -3.107 \rfloor = -4 \neq -3$, so FALSE.

b: Let us express the number $x = 0.\overline{215}$ as a fraction: $1000x = 215 + x$, so $999x = 215$, thus $x = \frac{215}{999}$. We get the equality, so FALSE. (The same method shows that, for example, $0.\overline{304} = \frac{304}{999}$ and $0.\overline{05} = \frac{5}{99}$.)

c: $f(x) = x\cos(\frac{3\pi}{2}+x) = x\sin x$, so $f(-x) = (-x)\cdot\sin(-x) = (-x)(-\sin x) = x\sin x = f(x)$. Thus f is even, TRUE.

10. Which of the given circles has bigger radius

$$x^2 - 6x + y^2 = 7 \quad \text{or} \quad x^2 + y^2 + 2y = 15\ ?$$

Complete squares: $(x-3)^2 - 9 + y^2 = 7$, $x^2 + (y+1)^2 - 1 = 15$. So, the equations are: $(x-3)^2 + y^2 = 16$, $x^2 + (y+1)^2 = 16$, thus radii are the same $r = 4$, the centres are points $(3, 0)$ and $(0, -1)$.