

MATH 251
WORKSHEET #2

1. For $f(x) = \frac{x+3}{5x+x^2+6}$ find

a: $\lim_{x \rightarrow -2^-} f(x)$, b: $\lim_{x \rightarrow -\infty} f(x)$, c: $\lim_{x \rightarrow -3} f(x)$,

find the horizontal and vertical asymptotes, and sketch the graph of $y = f(x)$.

2. For $g(x) = \frac{2-\sqrt{3x^2+1}}{x-1}$ find

a: $\lim_{x \rightarrow 1} g(x)$, b: $\lim_{x \rightarrow -\infty} g(x)$, c: $\lim_{x \rightarrow \infty} g(x)$.

3. Find the limits and state your answers as ∞ or $-\infty$ where appropriate.

a: $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - x - 6}$, b: $\lim_{x \rightarrow -\infty} (x^3 - 3x^5)$, c: $\lim_{x \rightarrow 1^-} \frac{x^2 - 2}{x^3 + 2x - 3}$,
d: $\lim_{x \rightarrow -3} \frac{x^3 + 27}{x^3 - 6x + 9}$, e: $\lim_{x \rightarrow 0} \frac{1 - |1 - 2x|}{x^2 - 2x}$, f: $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + 2x} - 3x)$.

4. a: Find $\lim_{x \rightarrow -\infty} \frac{3x + \sin(x + \frac{2}{x})}{x+3}$.

b: Suppose that $6x \leq f(x) \leq x^2 + 9$ for all x sufficiently close to 3, find $\lim_{x \rightarrow 3} f(x)$.

c: Suppose that $g(x) \geq \frac{\sqrt{x^2-3}}{x^2-x-2}$ for all $x > 2$, find $\lim_{x \rightarrow 2^+} g(x)$.

5. a: The Intermediate-Value Theorem allows us to conclude that the equation $\sqrt[3]{x+2} = 3-x$ has a root in the interval

a: $[-1, 0]$, b: $[0, 1]$, c: $[1, 2]$, d: $[2, 3]$, e: None of these.

b: Consider the equation $x^3 + 2x - 22 = 0$. Let r denote a root of the equation. Find an estimate for r with Error $< \frac{1}{4}$. **Hint:** Use the bisection method.

6. a: Let ε be a small positive real number. How close to 4 must we hold x to be sure that $\sqrt{x+5}$ lies within ε units of 3?

b: Let ε be a small positive real number, and let $x > 0$. How large must we hold x to be sure that $\frac{x}{x+2}$ lies within ε units of 1?

7. The function $\operatorname{sgn} x = \frac{x}{|x|}$ is neither continuous nor discontinuous at $x = 0$. How is this possible?