MATH 251
WORKSHEET #3

1. Differentiate the following functions:

(a) $x^3+2x^{\frac{5}{4}}-\dfrac{1}{x^3}-3,$ (b) $\dfrac{4}{\sqrt{x}}+5\sqrt{x}-\dfrac{1}{\sqrt[5]{x^3}},$ (c) $\dfrac{x}{x^2+1},$ (d) $\dfrac{3x+5}{4x-7},$

(e) $(x^3+2x-1)^{30},$ (f) $(x^3-x)\tan x,$ (g) $(2x+1)\sec x,$ (h) $\cos(4x^2+3x).$

2. For $f(x)=\dfrac{x}{x+2}, \quad g(x)=\dfrac{x+1}{x-1}$ find the compositions $g\circ g$ and $g\circ f$ and their domains.

3. The displacement of a particle at time t is given by $s(t)=t^3-t$. Find the average velocity over the time interval $1\le t\le 3$ and the instantaneous velocity when $t=3$.

4. The limit $\displaystyle\lim_{h\to 0}\frac{\sqrt{4+2h}-2}{h}$ represents the derivative of a certain function f at a certain point a. Which of the following is a possibility?

(a) $f(x)=\sqrt{x},\ a=2,$ (b) $f(x)=\sqrt{x},\ a=4,$ (c) $f(x)=\sqrt{2x},\ a=2,$

(d) $f(x)=2\sqrt{x},\ a=4,$ (e) None of these.

5. Let $f(x)=\sqrt{1-x}$.
   a: Use the Newton quotient to find $f'(x)$.
   b: Find the equation of the tangent line to the graph of f at $x=-3$.

6. Find the equation of the tangent line to the graph of
   a: $y=x^4-3x^3+2x^2+5x-4$ at the point $(1,1)$;
   b: $y=x^{1/2}-x^{-1/2}$ at the point $(1,0)$.

7. If f is differentiable at 2, $f(2)=-3$, $f'(2)=4$, find the equation of the line tangent to the graph of f at $x=2$.

8. Let $f(x)=x+\frac{12}{x^2}$. Find a point on the graph of f where the tangent line to f is parallel to the secant line which cuts the curve at $x=1$ and $x=2$.

9. Find all intervals where the function $f(x)=2x^3+3x^2-36x+4$ is increasing or decreasing.

10. If f is a differentiable even function then f' is

(a) even, (b) odd, (c) neither.

ANSWERS

1. (a) $3x^2 + \frac{5}{2}x^{\frac{1}{4}} + \frac{3}{x^4}$,
   (b) $-\frac{2}{\sqrt{x^3}} + \frac{5}{2\sqrt{x}} + \frac{3}{5\sqrt[5]{x^8}}$,
   (c) $\frac{1-x^2}{(1+x^2)^2}$,
   (d) $-\frac{41}{(4x-7)^2}$,
   (e) $30(x^3 + 2x - 1)^{29}(3x^2 + 2)$,
   (f) $(3x^2 - 1)\tan x + (x^3 - x)\sec^2 x$,
   (g) $2\sec x + (2x + 1)\sec x \tan x$,
   (h) $-(8x + 3)\sin(4x^2 + 3x)$.

2. $g \circ g(x) = x$ for $x \neq 1$, $D_{g\circ g} = (-\infty, 1) \cup (1, \infty)$;
   $g \circ f(x) = -x - 1$ for $x \neq -2$, $D_{g\circ f} = (-\infty, -2) \cup (-2, \infty)$.

3. Average velocity over the time interval $[1, 3]$ equals 12;
   instantaneous velocity at time $t = 3$ equals $s'(3) = 26$.

4. c.

5. a: $f'(x) = \lim_{h\to 0} \frac{\sqrt{1-x-h}-\sqrt{1-x}}{h} = -\frac{1}{2\sqrt{1-x}}$.
   b: $y = \frac{5-x}{4}$.

6. a: $y = 4x - 3$.
   b: $y = x - 1$.

7. $y = 4x - 11$.

8. $(\frac{2}{\sqrt[3]{3}}, \frac{11}{\sqrt[3]{3}})$.

9. f is increasing on $(-\infty, -3]$ and $[2, \infty)$, and decreasing on $[-3, 2]$.

10. b.