

MATH 251  
WORKSHEET #4

1. Find the third derivative  $f'''(x)$ , where  $f(x) = \frac{1}{x} + \sin x$ .
2. Find the equation of the tangent line to the graph of
  - a:  $\tan(xy) + \sin y = x^2y + 2x - 2$  at the point  $(1, 0)$ ;
  - b:  $xy^2 - x^2y = 16$  at the point  $(2, -2)$ .
3. Suppose that  $y$  is a differentiable function of  $x$  which satisfies the equation

$$(a) \ x^2y + xy^2 = 6, \quad (b) \ x^3 + y^3 = 27.$$

Find  $\frac{dy}{dx}$ . Suppose that  $y''$  exists. Find  $y''$ .

4. Find the indefinite integrals:

(a)  $\int x^{12} dx$

(b)  $\int \frac{x^2+4}{2x} dx$

(c)  $\int (x^2 + \sec^2 x + e^x) dx$

(d)  $\int 2x \sin(x^2) dx$

(e)  $\int (3 + x^{-4} - x^{-1} + e^{2x}) dx$

5. Find the solution  $y = y(x)$  to the given initial-value problem:

$$\begin{cases} y' = x^{1/3}, \\ y(0) = 5. \end{cases}$$

6. Suppose the position  $s(t)$  of a moving body is given by  $s(t) = te^{-t}$  for  $t \geq 0$  where  $t$  is the time.

(a) Find the velocity  $v(t)$  and the acceleration  $a(t)$ .

(b) Find all intervals where the body is speeding up and all intervals where it is slowing down.

7. Let  $g$  be the inverse function of  $f$ .

(a) Calculate  $g$  if  $f(x) = \frac{x}{1+x}$ . Specify the domains and ranges of  $f$  and  $g$ .

(b) Find  $g'(2)$  if  $f(x) = \frac{4x^3}{x^2+1}$ .

(c) Find  $g'(x)$  if  $f(x) = x\sqrt{3+x^2}$ .

8. Find the domain of each of the following functions:

(a)  $f(x) = \frac{e^x}{x}$

(b)  $g(x) = \sqrt{9-x^2}$

(c)  $h(x) = \ln(\ln x)$

9. Differentiate the following functions. If possible, simplify your answers.

(a)  $y = xe^x - x$ ,      (b)  $y = \ln |3x-2|$ ,      (c)  $y = \ln \ln x$ ,      (d)  $y = x^2 \ln(e^x+1)$ .

10. Simplify  $5e^{-\ln 5} - \ln(e^{-5})$ .