MATH 251
WORKSHEET #5

1. Simplify:

(a) $\log_{10} 1000 + \log_{10} 0.01$, (b) $(x^{-3})^{-2}$, (c) $2^{\log_4 8}$, (d) $\log_6 9 + \log_6 4$,
(e) $\tan(\arctan 200)$, (f) $\arccos(\sin(-0.2))$, (g) $\tan(\arcsin x)$.

2. Differentiate:

(a) $y = (\cos x)^x$, (b) $y = x^{\ln x}$, (c) $y = x \arctan x$, (d) $y = x^2 \arcsin x$.

3. Find the following limits:

(a) $\lim_{x\to 1^-} \log_x 2$, (b) $\lim_{x\to\infty} x^3 e^{-x}$.

4. The point P is moving along the circle $x^2+y^2 = 1m^2$ clockwise with angular velocity $3\, rad/s$. How fast is the distance of P from the point $Q = (0,2)m$ changing when $P = (1,0)m$?

5. Let $f(x) = x^3 + 3x^2 - 9x + 9$.
(a) Find all intervals where the function f is concave up or concave down.
(b) Find the (x, y) coordinates of all local maxima and minima, and inflexion points of f.

6. Sketch the graph of

(a) $y = \dfrac{x}{x^2-1}$, (b) $y = \dfrac{x^2}{x^2+1}$, (c) $y = \dfrac{x^2-4}{x+1}$,
(d) $y = \dfrac{\ln x}{x}$, (e) $y = x^2 e^x$.

Find any horizontal, vertical, or oblique asymptotes. Indicate points of all local maxima and minima, and inflexion points. Does the graph possess any symmetry?

7. (a) Among all rectangles of given area, show that the square has the least perimeter.
(b) A window has perimeter $10m$ and is in the shape of a rectangle with the top edge replaced by a semicircle. Find the dimensions of the rectangle if the window admits the greatest amount of light.

8. Let

(a) $f(x) = x^4 - 8x^2 - x + 16$, $x_0 = 2$; (b) $f(x) = x^3 - 997$, $x_0 = 10$.

Take the number x_0 as the first approximation for the root of $f(x) = 0$. Find the next approximation x_1, using the Newton's Method for Approximating Roots.

9. (a) Approximate $\sqrt[3]{997}$ using linearization.
(b) Determine the sign of the Error, and estimate its size.
(c) Specify an interval you can be sure contains $\sqrt[3]{997}$.