MATH 251
WORKSHEET #7 – SOLUTIONS

Final Quiz #6, April 14-15, covers sections
3.5 (The Inverse Trigonometric Functions + Right Triangle Rule, see worksheet #5),
4.4 (Graph Sketching, see worksheet #5 + quiz #4),
5.5 (The Fundamental Theorem of Calculus).

1. Find the integrals:

(a) $$\int_0^{\pi} \sin x \; dx = \int_0^{\pi} (-\cos x)' \; dx = (-\cos x)\Big|_0^{\pi} = (-\cos \pi) - (-\cos 0)$$
$$= -(-1) + 1 = 2.$$

(b) $$\int \frac{dx}{\cos^2 x} = \int \sec^2 x \; dx = \int (\tan x)' \; dx = \tan x + C.$$

(c) $$\int_{-\pi/4}^{\pi/4} \frac{dx}{\cos^2 x} = \int_{-\pi/4}^{\pi/4} \sec^2 x \; dx = \int_{-\pi/4}^{\pi/4} (\tan x)' \; dx = \tan x \Big|_{-\pi/4}^{\pi/4}$$
$$= \tan(\pi/4) - \tan(-\pi/4) = 1 - (-1) = 2.$$

(d) $$\int_a^b f'(g(x)) \; g'(x) \; dx = \int_a^b (f(g(x)))' \; dx = f(g(x))\Big|_a^b = f(g(b)) - f(g(a)).$$

2. Find the derivatives:

(a) $$\frac{d}{dx} \int_2^x \frac{\sin t}{t} dt = \frac{\sin t}{t}\Big|_{t=x} = \frac{\sin x}{x}.$$

(b) $$\frac{d}{dx} \int_x^3 \frac{\sin t}{t} dt = -\frac{d}{dx} \int_3^x \frac{\sin t}{t} dt = -\frac{\sin t}{t}\Big|_{t=x} = -\frac{\sin x}{x}.$$

(c) $$\frac{d}{dx} \int_0^{x^2} \frac{\cos t}{1+t^2} dt = (\text{by Chain Rule}) = \frac{d}{du} \left(\int_0^u \frac{\cos t}{1+t^2} dt \right)\Big|_{u=x^2} \cdot (x^2)'$$
$$= \frac{\cos t}{1+t^2}\Big|_{t=x^2} \cdot 2x = \frac{\cos(x^2)}{1+x^4} \cdot 2x = \frac{2x\cos(x^2)}{1+x^4}.$$