Practice Problems: S1 (Inequalities)

Solve the following inequalities for $x \in \mathbb{R}$. Write your solutions in interval notations.

1.
$$(3+2x)^2 \leq 4$$

2.
$$\frac{2x-1}{x+1} \geq \frac{2x+1}{x-1}$$

3.
$$3-2x < |3x-2|$$

4.
$$\frac{1}{|x-1|} < 2$$

5.
$$|2x+3| < |3x-7|$$

Recommended Problems: Pages 10-11 (Adams: Calculus, A Complete Course, 6th edition): Problems 23, 16, 20, 22, 26, 41, 42.

Solutions

1. $(3+2x)^2 \leq 4 \Leftrightarrow (3+2x)^2 - 4 \leq 0$

$$\Leftrightarrow (3+2x-2)(3+2x+2) \leq 0$$
$$\Leftrightarrow (1+2x)(5+2x) \leq 0$$
$$\Leftrightarrow x \in [-5/2, -1/2]$$

2. The inequality $\frac{2x-1}{x+1} \geq \frac{2x+1}{x-1}$ is defined for $x \in \mathbb{R} \setminus \{1,-1\}$.

For $x \in \mathbb{R} \setminus \{1,-1\}$, we have:

$$\frac{2x-1}{x+1} \geq \frac{2x+1}{x-1} \Leftrightarrow \frac{2x-1}{x+1} - \frac{2x+1}{x-1} \geq 0$$

$$\Leftrightarrow \frac{(2x-1)(x-1)-(2x+1)(x+1)}{(x+1)(x-1)} \geq 0$$

$$\Leftrightarrow \frac{2x^2-2x-x+1-(2x^2+2x+x+1)}{(x+1)(x-1)} \geq 0$$

$$\Leftrightarrow \frac{-6x}{(x+1)(x-1)} \geq 0 \Leftrightarrow \frac{6x}{(x+1)(x-1)} \leq 0$$

x	$-\infty$		-1		0		1		$+\infty$
x		$-$		$-$	0	$+$		$+$	
$x+1$		$-$	0	$+$		$+$		$+$	
$x-1$		$-$		$-$		$-$	0	$+$	
$\frac{6x}{(x+1)(x-1)}$		$-$	$\Vert$	$+$	0	$-$	$\Vert$	$+$	

So, $S = (-\infty,-1) \cup [0,1)$ is the solution set

3. $3-2x<|3x-2|$

$$|3x-2| = \begin{cases} 3x-2 & \text{if } 3x-2 \geq 0 \iff x \in [2/3, +\infty) \\ -3x+2 & \text{if } 3x-2<0 \iff x \in (-\infty, 2/3) \end{cases}$$

- If $\underline{x \geq \frac{2}{3}}$, i.e. $x \in [2/3, +\infty)$, then $|3x-2| = 3x-2$. We have

$3-2x < 3x-2 \iff 5<5x \iff \underline{1<x}$ or $x \in (1, +\infty)$

So, $S_1 = [2/3, +\infty) \cap (1, +\infty) = (1, +\infty)$

- If $\underline{x < 2/3}$, then $|3x-2| = -3x+2$. We have

$3-2x < -3x+2 \iff x<-1$

So, $S_2 = (-\infty, 2/3) \cap (-\infty, -1) = (-\infty, -1)$

Finally, the solution set to the inequality is

$\underline{S = S_1 \cup S_2} = (1, +\infty) \cup (-\infty, -1)$

4. $\frac{1}{|x-1|} < 2$ is defined for $x \neq 1$.

For $x \neq 1$, $|x-1| > 0$, so $\frac{1}{|x-1|} < 2 \iff 1 < 2|x-1|$

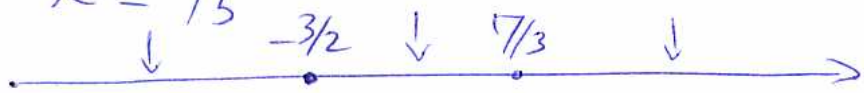
$\iff \frac{1}{2} < |x-1| \iff \frac{1}{2} < x-1$ or $x-1 < -1/2$

$\iff 3/2 < x$ or $x < 1/2$, i.e.

$x \in S = (-\infty, 1/2) \cup (3/2, +\infty)$

5. $|2x+3| < |3x-7|$

There are two "critical" points: $x = -3/2$ and $x = 7/3$.



We consider three cases:

1°) $x < -3/2$; 2°) $-3/2 \le x < 7/3$; 3°) $x \geq 7/3$.

<u>Case 1</u>: $\underline{x < -3/2}$ i.e. $x \in (-\infty, -3/2)$

In this case, we have

$2x+3 < 0$ and $3x-7 < 0$ $(x < -3/2 < 7/3)$

$\Rightarrow |2x+3| = -2x-3$ and $|3x-7| = -3x+7$

The inequality becomes:

$-2x-3 < -3x+7 \iff \underline{x < 10}$

$\Rightarrow S_1 = (-\infty, -3/2) \cap (-\infty, 10) = (-\infty, -3/2)$ is the first part of the solution set

<u>Case 2</u>: $\underline{-3/2 \le x < 7/3}$ i.e. $x \in [-3/2, 7/3)$

$\Rightarrow 2x+3 \geq 0$ and $3x-7 < 0$

$\Rightarrow |2x+3| = 2x+3$ and $|3x-7| = -3x+7$

So $2x+3 < -3x+7 \iff 5x < 4 \iff \underline{x < 4/5}$

$S_2 = [-3/2, 7/3) \cap (-\infty, 4/5) = [-3/2, 4/5)$ (second part