

APPENDIX A

Real Numbers, Intervals, and Inequalities

EXERCISE SET A

1. (a) rational (b) integer, rational (c) integer, rational
 (d) rational (e) integer, rational (f) irrational
 (g) rational (h) 467 integer, rational
2. (a) irrational (b) rational (c) rational (d) rational
3. (a) $x = 0.123123123\dots$, $1000x = 123 + x$, $x = 123/999 = 41/333$
 (b) $x = 12.7777\dots$, $10(x - 12) = 7 + (x - 12)$, $9x = 115$, $x = 115/9$
 (c) $x = 38.07818181\dots$, $100x = 3807.81818181\dots$, $99x = 100x - x = 3769.74$,

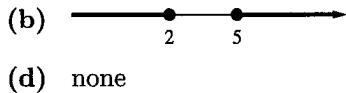
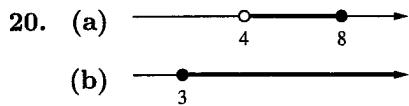
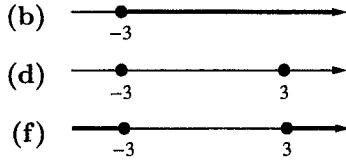
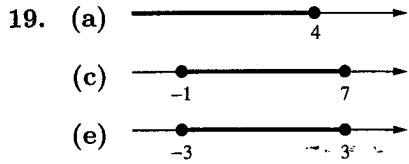
$$x = \frac{3769.74}{99} = \frac{376974}{9900} = \frac{20943}{550}$$

 (d) $\frac{4296}{10000} = \frac{537}{1250}$
4. $x = 0.99999\dots$, $10x = 9 + x$, $9x = 9$, $x = 1$
5. (a) If r is the radius, then $D = 2r$ so $\left(\frac{8}{9}D\right)^2 = \left(\frac{16}{9}r\right)^2 = \frac{256}{81}r^2$. The area of a circle of radius r is πr^2 so $256/81$ was the approximation used for π .
 (b) $22/7 \approx 3.1429$ is better than $256/81 \approx 3.1605$.
6. (a) $\frac{223}{71} < \frac{333}{106} < \frac{63}{25} \left(\frac{17 + 15\sqrt{5}}{7 + 15\sqrt{5}} \right) < \frac{355}{113} < \frac{22}{7}$
 (b) Ramanujan's (c) Athoniszoon's (d) Ramanujan's
7.

Line	2	3	4	5	6	7
Blocks	3, 4	1, 2	3, 4	2, 4, 5	1, 2	3, 4
8.

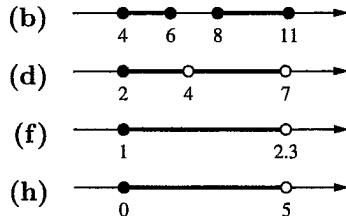
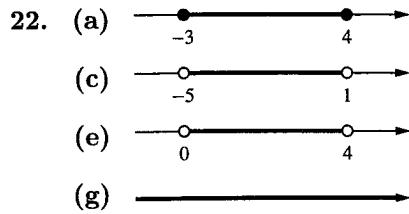
Line	1	2	3	4	5
Blocks	all blocks	none	2, 4	2	2, 3
9. (a) always correct (add -3 to both sides of $a \leq b$)
 (b) not always correct (correct only if $a = b$)
 (c) not always correct (correct only if $a = b$)
 (d) always correct (multiply both sides of $a \leq b$ by 6)
 (e) not always correct (correct only if $a \geq 0$ or $a = b$)
 (f) always correct (multiply both sides of $a \leq b$ by the nonnegative quantity a^2)
10. (a) always correct
 (b) not always correct (for example let $a = b = 0$, $c = 1$, $d = 2$)
 (c) not always correct (for example let $a = 1$, $b = 2$, $c = d = 0$)
11. (a) all values because $a = a$ is always valid (b) none
12. $a = b$, because if $a \neq b$ then $a < b$ and $b < a$ are contradictory

13. (a) yes, because $a \leq b$ means $a < b$ or $a = b$, thus $a < b$ certainly means $a \leq b$
 (b) no, because $a < b$ is false if $a = b$ is true
14. (a) $x^2 - 5x = 0$, $x(x - 5) = 0$ so $x = 0$ or $x = 5$
 (b) $-1, 0, 1, 2$ are the only integers that satisfy $-2 < x < 3$
15. (a) $\{x : x \text{ is a positive odd integer}\}$ (b) $\{x : x \text{ is an even integer}\}$
 (c) $\{x : x \text{ is irrational}\}$ (d) $\{x : x \text{ is an integer and } 7 \leq x \leq 10\}$
16. (a) not equal to A because 0 is not in A (b) equal to A
 (c) equal to A because $(x - 3)(x^2 - 3x + 2) = 0$, $(x - 3)(x - 2)(x - 1) = 0$ so $x = 1, 2$, or 3
17. (a) false, there are points inside the triangle that are not inside the circle
 (b) true, all points inside the triangle are also inside the square
 (c) true (d) false (e) true
 (f) true, a is inside the circle (g) true
18. (a) $\emptyset, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}$ (b) \emptyset



21. (a) $[-2, 2]$

(b) $(-\infty, -2) \cup (2, +\infty)$



23. $3x < 10$; $(-\infty, 10/3)$

24. $\frac{1}{5}x \geq 8$; $[40, +\infty)$

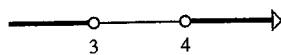
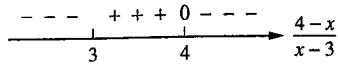
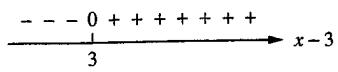
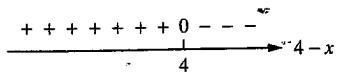
25. $2x \leq -11$; $(-\infty, -11/2]$

26. $9x < -10$; $(-\infty, -10/9)$

27. $2x \leq 1$ and $2x > -3$; $(-3/2, 1/2]$

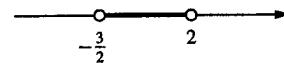
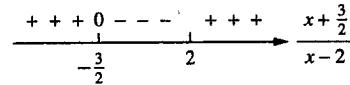
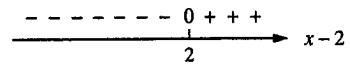
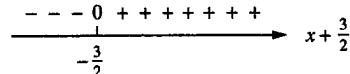
28. $8x \geq 5$ and $8x \leq 14$; $[\frac{5}{8}, \frac{7}{4}]$

29. $\frac{x}{x-3} - 4 < 0, \frac{12-3x}{x-3} < 0, \frac{4-x}{x-3} < 0;$
 $(-\infty, 3) \cup (4, +\infty)$

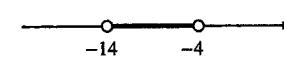
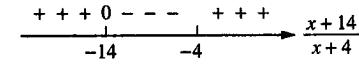
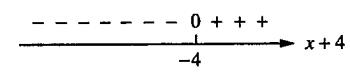
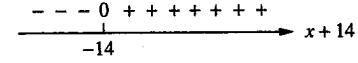


31. $\frac{3x+1}{x-2} - 1 = \frac{2x+3}{x-2} < 0, \frac{x+3/2}{x-2} < 0;$
 $(-\frac{3}{2}, 2)$

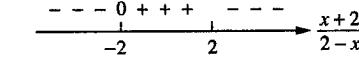
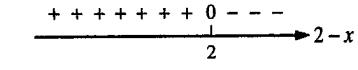
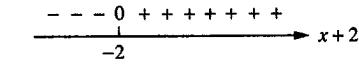
→ → → → →



32. $\frac{x/2-3}{4+x} - 1 > 0, \frac{x-6}{4+x} - 2 > 0, \frac{x+14}{x+4} < 0;$
 $(-14, -4)$



33. $\frac{4}{2-x} - 1 = \frac{x+2}{2-x} \leq 0; (-\infty, -2] \cup (2, +\infty)$

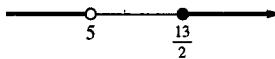


34. $\frac{3}{x-5} - 2 = \frac{13-2x}{x-5} \leq 0$, $\frac{13/2-x}{x-5} \leq 0$;
 $(-\infty, 5) \cup [\frac{13}{2}, +\infty)$

$$\begin{array}{c} + + + + + + 0 - - - \\ \hline & & & & & & 13 \\ & & & & & & \hline & & & & & & \frac{13}{2} - x \\ & & & & & & \hline & & & & & & \frac{13}{2} \\ & & & & & & \hline \end{array}$$

$$\begin{array}{c} - - - 0 + + + + + + \\ \hline & & & & & & 1 \\ & & & & & & \hline & & & & & & 5 \\ & & & & & & \hline \end{array}$$

$$\begin{array}{c} - - - + + + 0 - - - \\ \hline 5 & & & & & & 13 \\ & & & & & & \hline & & & & & & \frac{13}{2} - x \\ & & & & & & \hline & & & & & & \frac{13}{2} \\ & & & & & & \hline \end{array}$$

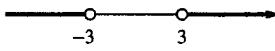


35. $x^2 - 9 = (x+3)(x-3) > 0$;
 $(-\infty, -3) \cup (3, +\infty)$

$$\begin{array}{c} - - - 0 + + + + + + \\ \hline & & & & & & 1 \\ & & & & & & \hline & & & & & & -3 \\ & & & & & & \hline \end{array}$$

$$\begin{array}{c} - - - - - 0 + + + \\ \hline & & & & & & 1 \\ & & & & & & \hline & & & & & & 3 \\ & & & & & & \hline \end{array}$$

$$\begin{array}{c} + + + 0 - - - 0 + + + \\ \hline -3 & & & & & & 1 \\ & & & & & & \hline & & & & & & (x+3)(x-3) \\ & & & & & & \hline & & & & & & \end{array}$$



36. $x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5}) \leq 0$; $[-\sqrt{5}, \sqrt{5}]$

$$\begin{array}{c} - - - 0 + + + + + + \\ \hline & & & & & & 1 \\ & & & & & & \hline & & & & & & -\sqrt{5} \\ & & & & & & \hline \end{array}$$

$$\begin{array}{c} - - - - - 0 + + + \\ \hline & & & & & & 1 \\ & & & & & & \hline & & & & & & \sqrt{5} \\ & & & & & & \hline \end{array}$$

$$\begin{array}{c} + + + 0 - - - 0 + + + \\ \hline -\sqrt{5} & & & & & & 1 \\ & & & & & & \hline & & & & & & (x+\sqrt{5})(x-\sqrt{5}) \\ & & & & & & \hline & & & & & & \end{array}$$

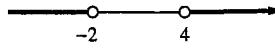


37. $(x-4)(x+2) > 0$; $(-\infty, -2) \cup (4, +\infty)$

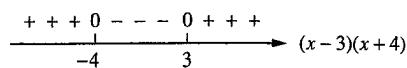
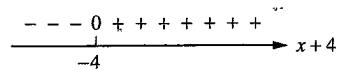
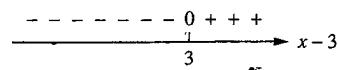
$$\begin{array}{c} - - - - - 0 + + + \\ \hline & & & & & & 1 \\ & & & & & & \hline & & & & & & 4 \\ & & & & & & \hline \end{array}$$

$$\begin{array}{c} - - - 0 + + + + + + \\ \hline & & & & & & 1 \\ & & & & & & \hline & & & & & & -2 \\ & & & & & & \hline \end{array}$$

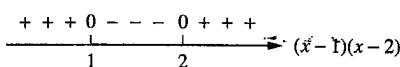
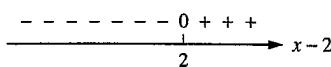
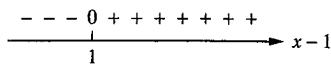
$$\begin{array}{c} + + + 0 - - - 0 + + + \\ \hline -2 & & & & & & 1 \\ & & & & & & \hline & & & & & & (x-4)(x+2) \\ & & & & & & \hline & & & & & & \end{array}$$



38. $(x - 3)(x + 4) < 0; (-4, 3)$

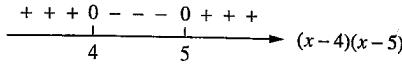
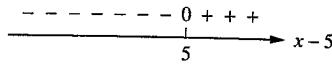
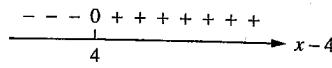


40. $(x - 2)(x - 1) \geq 0;$
 $(-\infty, 1] \cup [2, +\infty)$



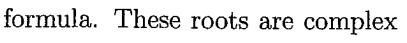
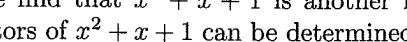
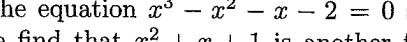
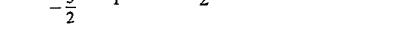
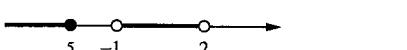
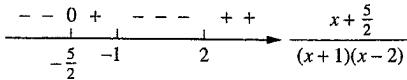
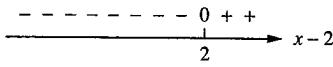
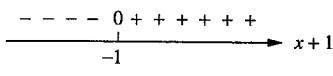
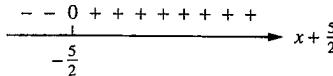
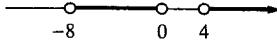
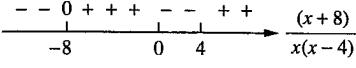
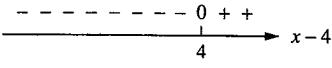
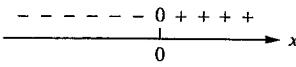
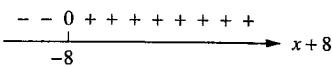
42. $\frac{1}{x+1} - \frac{3}{x-2} = \frac{-2x-5}{(x+1)(x-2)} \geq 0,$
 $\frac{x+5/2}{(x+1)(x-2)} \leq 0;$
 $(-\infty, -\frac{5}{2}] \cup (-1, 2)$

39. $(x - 4)(x - 5) \leq 0; [4, 5]$



41. $\frac{3}{x-4} - \frac{2}{x} = \frac{x+8}{x(x-4)} > 0;$

$(-8, 0) \cup (4, +\infty)$



43. By trial-and-error we find that $x = 2$ is a root of the equation $x^3 - x^2 - x - 2 = 0$ so $x - 2$ is a factor of $x^3 - x^2 - x - 2$. By long division we find that $x^2 + x + 1$ is another factor so $x^3 - x^2 - x - 2 = (x - 2)(x^2 + x + 1)$. The linear factors of $x^2 + x + 1$ can be determined by first finding the roots of $x^2 + x + 1 = 0$ by the quadratic formula. These roots are complex numbers

so $x^2 + x + 1 \neq 0$ for all real x ; thus $x^2 + x + 1$ must be always positive or always negative. Since $x^2 + x + 1$ is positive when $x = 0$, it follows that $x^2 + x + 1 > 0$ for all real x . Hence $x^3 - x^2 - x - 2 > 0$, $(x - 2)(x^2 + x + 1) > 0$, $x - 2 > 0$, $x > 2$, so $S = (2, +\infty)$.

44. By trial-and-error we find that $x = 1$ is a root of the equation $x^3 - 3x + 2 = 0$ so $x - 1$ is a factor of $x^3 - 3x + 2$. By long division we find that $x^2 + x - 2$ is another factor so $x^3 - 3x + 2 = (x - 1)(x^2 + x - 2) = (x - 1)(x - 1)(x + 2) = (x - 1)^2(x + 2)$. Therefore we want to solve $(x - 1)^2(x + 2) \leq 0$. Now if $x \neq 1$, then $(x - 1)^2 > 0$ and so $x + 2 \leq 0$, $x \leq -2$. By inspection, $x = 1$ is also a solution so $S = (-\infty, -2] \cup \{1\}$.
45. $\sqrt{x^2 + x - 6}$ is real if $x^2 + x - 6 \geq 0$. Factor to get $(x + 3)(x - 2) \geq 0$ which has as its solution $x \leq -3$ or $x \geq 2$.
46. $\frac{x+2}{x-1} \geq 0$; $(-\infty, -2] \cup (1, +\infty)$
47. $25 \leq \frac{5}{9}(F - 32) \leq 40$, $45 \leq F - 32 \leq 72$, $77 \leq F \leq 104$
48. (a) $n = 2k$, $n^2 = 4k^2 = 2(2k^2)$ where $2k^2$ is an integer.
 (b) $n = 2k + 1$, $n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$ where $2k^2 + 2k$ is an integer.
49. (a) Assume m and n are rational, then $m = \frac{p}{q}$ and $n = \frac{r}{s}$ where p, q, r , and s are integers so $m + n = \frac{p}{q} + \frac{r}{s} = \frac{ps + rq}{qs}$ which is rational because $ps + rq$ and qs are integers.
 (b) (proof by contradiction) Assume m is rational and n is irrational, then $m = \frac{p}{q}$ where p and q are integers. Suppose that $m + n$ is rational, then $m + n = \frac{r}{s}$ where r and s are integers so $n = \frac{r}{s} - m = \frac{r}{s} - \frac{p}{q} = \frac{rq - ps}{sq}$. But $rq - ps$ and sq are integers, so n is rational which contradicts the assumption that n is irrational.
50. (a) Assume m and n are rational, then $m = \frac{p}{q}$ and $n = \frac{r}{s}$ where p, q, r , and s are integers so $mn = \frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}$ which is rational because pr and qs are integers.
 (b) (proof by contradiction) Assume m is rational and nonzero and that n is irrational, then $m = \frac{p}{q}$ where p and q are integers and $p \neq 0$. Suppose that mn is rational, then $mn = \frac{r}{s}$ where r and s are integers so $n = \frac{r/s}{m} = \frac{r/s}{p/q} = \frac{rq}{ps}$. But rq and ps are integers, so n is rational which contradicts the assumption that n is irrational.
51. $a = \sqrt{2}$, $b = \sqrt{3}$, $c = \sqrt{6}$, $d = -\sqrt{2}$ are irrational, and $a + d = 0$, a rational; $a + a = 2\sqrt{2}$, an irrational; $ad = -2$, a rational; and $ab = c$, an irrational.
52. (a) irrational (Exercise 49(b))
 (b) irrational (Exercise 50(b))
 (c) rational by inspection; Exercise 51 gives no information
 (d) $\sqrt{\pi}$ must be irrational, for if it were rational, then so would be $\pi = (\sqrt{\pi})^2$ by Exercise 50(a); but π is known to be irrational.
53. The average of a and b is $\frac{1}{2}(a + b)$; if a and b are rational then so is the average, by Exercise 49(a) and Exercise 50(a). On the other hand if $a = b = \sqrt{2}$ then the average of a and b is irrational, but the average of a and $-b$ is rational.

54. If $10^x = 3$, then $x > 0$ because $10^x \leq 1$ for $x \leq 0$. If $10^{p/q} = 3$ with p, q integers, then $10^p = 3^q$. Following Exercise 48, if $n = 2k$ is even, then n^2, n^3, n^4, \dots are even; and if $n = 2k + 1$ then n^2, n^3, n^4, \dots are odd. Since $10^p = 3^q$, the left side is even and the right side is odd, a contradiction.
55. $8x^3 - 4x^2 - 2x + 1$ can be factored by grouping terms:
 $(8x^3 - 4x^2) - (2x - 1) = 4x^2(2x - 1) - (2x - 1) = (2x - 1)(4x^2 - 1) = (2x - 1)^2(2x + 1)$. The problem, then, is to solve $(2x - 1)^2(2x + 1) < 0$. By inspection, $x = 1/2$ is not a solution. If $x \neq 1/2$, then $(2x - 1)^2 > 0$ and it follows that $2x + 1 < 0$, $2x < -1$, $x < -1/2$, so $S = (-\infty, -1/2)$.
56. Rewrite the inequality as $12x^3 - 20x^2 + 11x - 2 \geq 0$. If a polynomial in x with integer coefficients has a rational zero $\frac{p}{q}$, a fraction in lowest terms, then p must be a factor of the constant term and q must be a factor of the coefficient of the highest power of x . By trial-and-error we find that $x = 1/2$ is a zero, thus $(x - 1/2)$ is a factor so

$$\begin{aligned} 12x^3 - 20x^2 + 11x - 2 &= (x - 1/2)(12x^2 - 14x + 4) \\ &= 2(x - 1/2)(6x^2 - 7x + 2) \\ &= 2(x - 1/2)(2x - 1)(3x - 2) = (2x - 1)^2(3x - 2). \end{aligned}$$

Now to solve $(2x - 1)^2(3x - 2) \geq 0$ we first note that $x = 1/2$ is a solution. If $x \neq 1/2$ then $(2x - 1)^2 > 0$ and $3x - 2 \geq 0$, $3x \geq 2$, $x \geq 2/3$ so $S = [2/3, +\infty) \cup \{1/2\}$.

57. If $a < b$, then $ac < bc$ because c is positive; if $c < d$, then $bc < bd$ because b is positive, so $ac < bd$ (Theorem A.1(a)). (Note that the result is still true if one of a, b, c, d is allowed to be negative, that is $a < 0$ or $c < 0$.)
58. no, since the decimal representation is not repeating (the string of zeros does not have constant length)

APPENDIX B

Absolute Value

EXERCISE SET B

21. $|9x| - 11 = x$

<u>Case 1:</u>	<u>Case 2:</u>
$9x - 11 = x$	$-9x - 11 = x$
$8x = 11$	$-10x = 11$
$x = 11/8$	$x = -11/10$

22. $2x - 7 = |x + 1|$

<u>Case 1:</u>	<u>Case 2:</u>
$2x - 7 = x + 1$	$2x - 7 = -(x + 1)$
$x = 8$	$3x = 6$
$x = 2$; not a solution because x must also satisfy $x < -1$	

23. $\left| \frac{x+5}{2-x} \right| = 6$

<u>Case 1:</u>	<u>Case 2:</u>
$\frac{x+5}{2-x} = 6$	$\frac{x+5}{2-x} = -6$
$x+5 = 12 - 6x$	$x+5 = -12 + 6x$
$7x = 7$	$-5x = -17$
$x = 1$	$x = 17/5$

24. $\left| \frac{x-3}{x+4} \right| = 5$

<u>Case 1:</u>	<u>Case 2:</u>
$\frac{x-3}{x+4} = 5$	$\frac{x-3}{x+4} = -5$
$x-3 = 5x + 20$	$x-3 = -5x - 20$
$-4x = 23$	$6x = -17$
$x = -23/4$	$x = -17/6$

25. $|x + 6| < 3$

$$\begin{aligned} -3 &< x + 6 < 3 \\ -9 &< x < -3 \end{aligned}$$

$$S = (-9, -3)$$

26. $|7 - x| \leq 5$

$$\begin{aligned} -5 &\leq 7 - x \leq 5 \\ -12 &\leq -x \leq -2 \\ 12 &\geq x \geq 2 \end{aligned}$$

$$S = [2, 12]$$

27. $|2x - 3| \leq 6$

$$\begin{aligned} -6 &\leq 2x - 3 \leq 6 \\ -3 &\leq 2x \leq 9 \\ -3/2 &\leq x \leq 9/2 \end{aligned}$$

$$S = [-3/2, 9/2]$$

28. $|3x + 1| < 4$

$$\begin{aligned} -4 &< 3x + 1 < 4 \\ -5 &< 3x < 3 \\ -5/3 &< x < 1 \end{aligned}$$

$$S = (-5/3, 1)$$

29. $|x + 2| > 1$

<u>Case 1:</u>	<u>Case 2:</u>
$x + 2 > 1$	$x + 2 < -1$
$x > -1$	$x < -3$
$S = (-\infty, -3) \cup (-1, +\infty)$	

30. $\left| \frac{1}{2}x - 1 \right| \geq 2$

<u>Case 1:</u>	<u>Case 2:</u>
$\frac{1}{2}x - 1 \geq 2$	$\frac{1}{2}x - 1 \leq -2$
$\frac{1}{2}x \geq 3$	$\frac{1}{2}x \leq -1$
$x \geq 6$	$x \leq -2$
$S = (-\infty, -2] \cup [6, +\infty)$	

31. $|5 - 2x| \geq 4$

<u>Case 1:</u>	<u>Case 2:</u>
$5 - 2x \geq 4$	$5 - 2x \leq -4$
$-2x \geq -1$	$-2x \leq -9$
$x \leq 1/2$	$x \geq 9/2$
$S = (-\infty, 1/2] \cup [9/2, +\infty)$	

32. $|7x + 1| > 3$

<u>Case 1:</u>	<u>Case 2:</u>
$7x + 1 > 3$	$7x + 1 < -3$
$7x > 2$	$7x < -4$
$x > 2/7$	$x < -4/7$
$S = (-\infty, -4/7) \cup (2/7, +\infty)$	

33. $\frac{1}{|x-1|} < 2, x \neq 1$

$$|x-1| > 1/2$$

Case 1: Case 2:

$$x-1 > 1/2 \quad x-1 < -1/2$$

$$x > 3/2 \quad x < 1/2$$

$$S = (-\infty, 1/2) \cup (3/2, +\infty)$$

34. $\frac{1}{|3x+1|} \geq 5, x \neq -1/3$

$$|3x+1| \leq 1/5$$

$$-1/5 \leq 3x+1 \leq 1/5$$

$$-6/5 \leq 3x \leq -4/5$$

$$-2/5 \leq x \leq -4/15$$

$$S = [-2/5, -1/3] \cup (-1/3, -4/15]$$

35. $\frac{3}{|2x-1|} \geq 4, x \neq 1/2$

$$\frac{|2x-1|}{3} \leq \frac{1}{4}$$

$$|2x-1| \leq 3/4$$

$$-3/4 \leq 2x-1 \leq 3/4$$

$$1/4 \leq 2x \leq 7/4$$

$$1/8 \leq x \leq 7/8$$

$$S = [1/8, 1/2) \cup (1/2, 7/8]$$

36. $\frac{2}{|x+3|} < 1, x \neq -3$

$$\frac{|x+3|}{2} > 1$$

$$|x+3| > 2$$

Case 1: Case 2:

$$x+3 > 2 \quad x+3 < -2$$

$$x > -1 \quad x < -5$$

$$S = (-\infty, -5) \cup (-1, +\infty)$$

37. $\sqrt{(x^2 - 5x + 6)^2} = x^2 - 5x + 6$ if $x^2 - 5x + 6 \geq 0$ or, equivalently, if $(x-2)(x-3) \geq 0$;
 $x \in (-\infty, 2] \cup [3, +\infty)$.

38. If $x \geq 2$ then $3 \leq x-2 \leq 7$ so $5 \leq x \leq 9$; if $x < 2$ then $3 \leq 2-x \leq 7$ so $-5 \leq x \leq -1$.
 $S = [-5, -1] \cup [5, 9]$.

39. If $u = |x-3|$ then $u^2 - 4u = 12$, $u^2 - 4u - 12 = 0$, $(u-6)(u+2) = 0$, so $u = 6$ or $u = -2$. If $u = 6$ then $|x-3| = 6$, so $x = 9$ or $x = -3$. If $u = -2$ then $|x-3| = -2$ which is impossible. The solutions are -3 and 9 .

41. $|a-b| = |a+(-b)|$
 $\leq |a| + |-b|$ (triangle inequality)
 $= |a| + |b|$.

42. $a = (a-b) + b$
 $|a| = |(a-b) + b|$
 $|a| \leq |a-b| + |b|$ (triangle inequality)
 $|a| - |b| \leq |a-b|$.

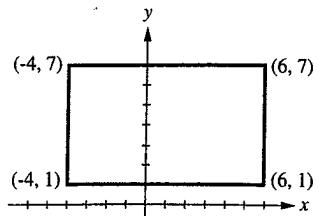
43. From Exercise 42
(i) $|a| - |b| \leq |a-b|$; but $|b| - |a| \leq |b-a| = |a-b|$, so (ii) $|a| - |b| \geq -|a-b|$.
Combining (i) and (ii): $-|a-b| \leq |a| - |b| \leq |a-b|$, so $||a| - |b|| \leq |a-b|$.

APPENDIX C

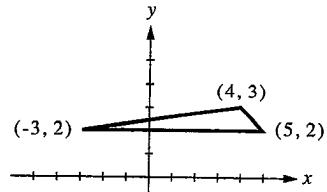
Coordinate Planes and Lines

EXERCISE SET C

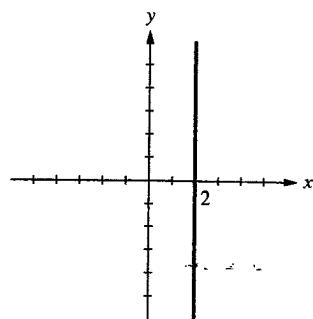
1.



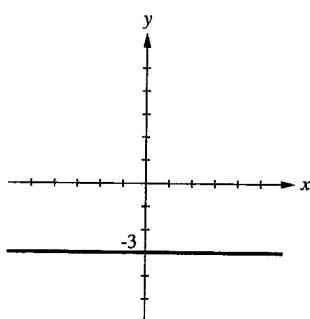
2. area = $\frac{1}{2}bh = \frac{1}{2}(5 - (-3))(1) = 4$



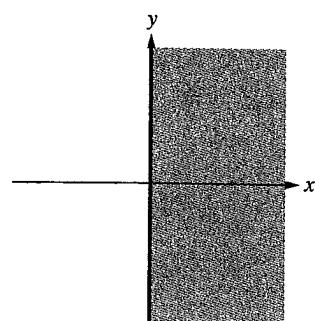
3. (a) $x = 2$



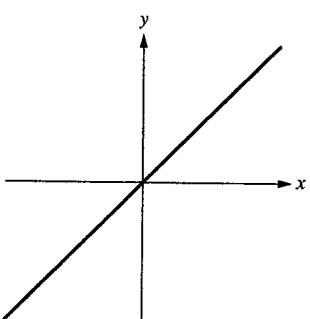
(b) $y = -3$



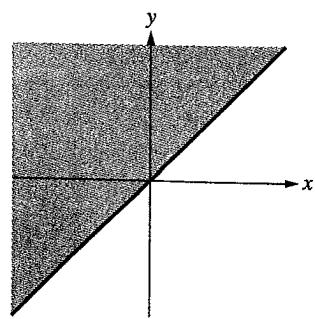
(c) $x \geq 0$



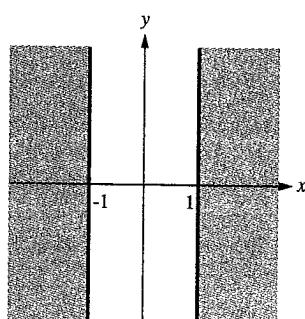
(d) $y = x$



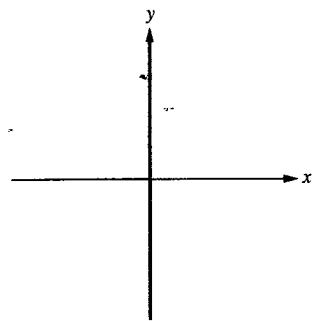
(e) $y \geq x$



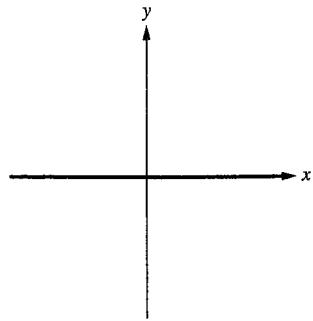
(f) $|x| \geq 1$



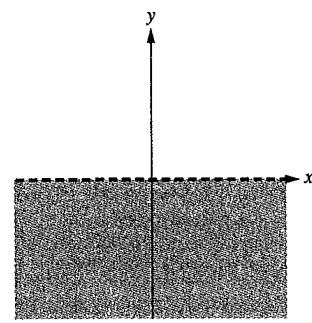
4. (a) $x = 0$



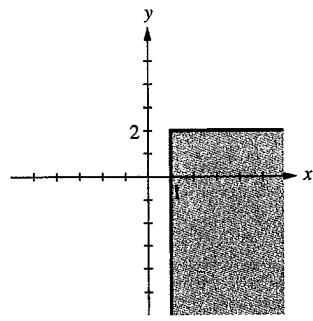
(b) $y = 0$



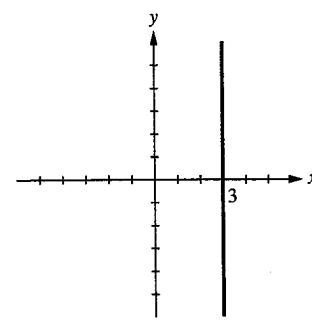
(c) $y < 0$



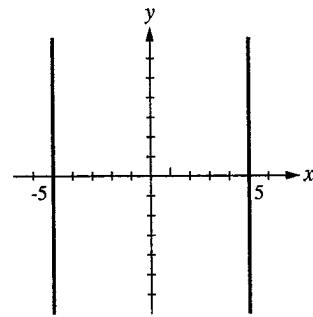
(d) $x \geq 1$ and $y \leq 2$



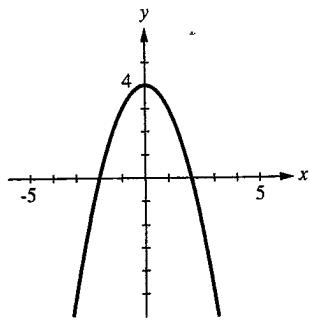
(e) $x = 3$



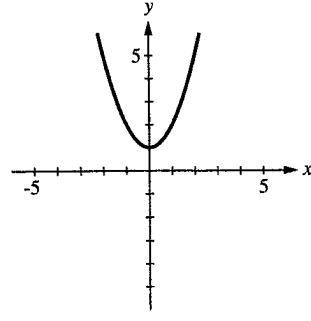
(f) $|x| = 5$



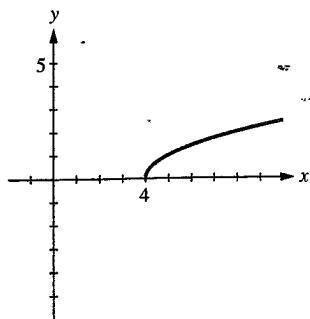
5. $y = 4 - x^2$



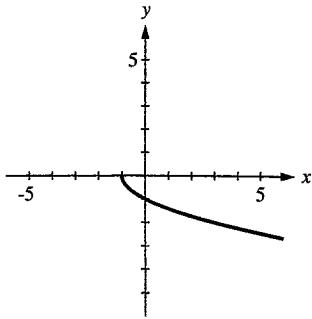
6. $y = 1 + x^2$



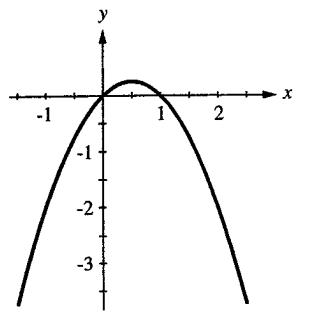
7. $y = \sqrt{x - 4}$



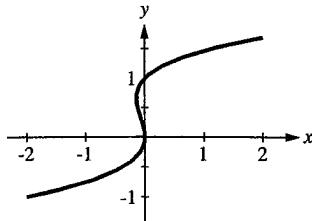
8. $y = -\sqrt{x + 1}$



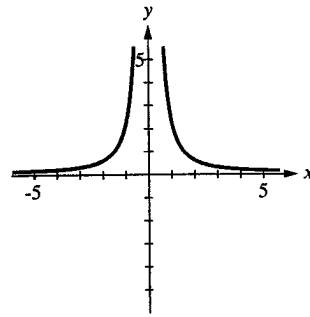
9. $x^2 - x + y = 0$



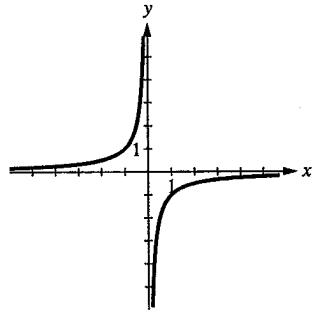
10. $x = y^3 - y^2$



11. $x^2y = 2$



12. $xy = -1$



13. (a) $m = \frac{4 - 2}{3 - (-1)} = \frac{1}{2}$

(b) $m = \frac{1 - 3}{7 - 5} = -1$

(c) $m = \frac{\sqrt{2} - \sqrt{2}}{-3 - 4} = 0$

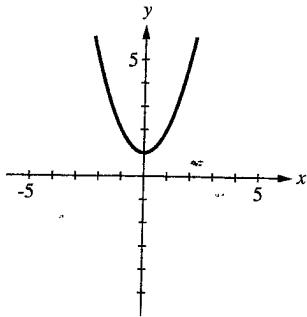
(d) $m = \frac{12 - (-6)}{-2 - (-2)} = \frac{18}{0}$, not defined

14. $m_1 = \frac{5 - 2}{6 - (-1)} = \frac{3}{7}$, $m_2 = \frac{7 - 2}{2 - (-1)} = \frac{5}{3}$, $m_3 = \frac{7 - 5}{2 - 6} = -\frac{1}{2}$

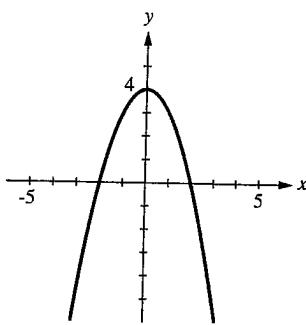
15. (a) The line through $(1, 1)$ and $(-2, -5)$ has slope $m_1 = \frac{-5 - 1}{-2 - 1} = 2$, the line through $(1, 1)$ and $(0, -1)$ has slope $m_2 = \frac{-1 - 1}{0 - 1} = 2$. The given points lie on a line because $m_1 = m_2$.

(b) The line through $(-2, 4)$ and $(0, 2)$ has slope $m_1 = \frac{2 - 4}{0 + 2} = -1$, the line through $(-2, 4)$ and $(1, 5)$ has slope $m_2 = \frac{5 - 4}{1 + 2} = \frac{1}{3}$. The given points do not lie on a line because $m_1 \neq m_2$.

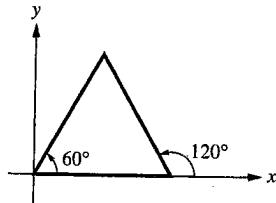
16.



17.



18. The triangle is equiangular because it is equilateral. The angles of inclination of the sides are 0° , 60° , and 120° (see figure), thus the slopes of its sides are $\tan 0^\circ = 0$, $\tan 60^\circ = \sqrt{3}$, and $\tan 120^\circ = -\sqrt{3}$.

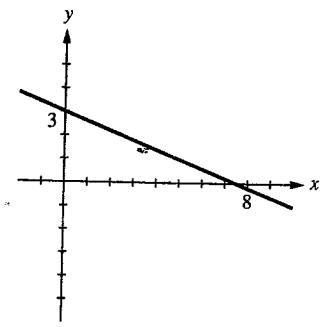


19. III < II < IV < I

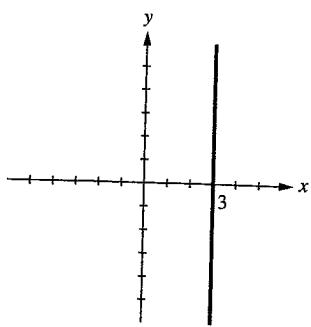
20. III < IV < I < II

21. Use the points $(1, 2)$ and (x, y) to calculate the slope: $(y - 2)/(x - 1) = 3$
- if $x = 5$, then $(y - 2)/(5 - 1) = 3$, $y - 2 = 12$, $y = 14$
 - if $y = -2$, then $(-2 - 2)/(x - 1) = 3$, $x - 1 = -4/3$, $x = -1/3$
22. Use $(7, 5)$ and (x, y) to calculate the slope: $(y - 5)/(x - 7) = -2$
- if $x = 9$, then $(y - 5)/(9 - 7) = -2$, $y - 5 = -4$, $y = 1$
 - if $y = 12$, then $(12 - 5)/(x - 7) = -2$, $x - 7 = -7/2$, $x = 7/2$
23. Using $(3, k)$ and $(-2, 4)$ to calculate the slope, we find $\frac{k - 4}{3 - (-2)} = 5$, $k - 4 = 25$, $k = 29$.
24. The slope obtained by using the points $(1, 5)$ and $(k, 4)$ must be the same as that obtained from the points $(1, 5)$ and $(2, -3)$ so $\frac{4 - 5}{k - 1} = \frac{-3 - 5}{2 - 1}$, $-\frac{1}{k - 1} = -8$, $k - 1 = 1/8$, $k = 9/8$.
25. $\frac{0 - 2}{x - 1} = -\frac{0 - 5}{x - 4}$, $-2x + 8 = 5x - 5$, $7x = 13$, $x = 13/7$
26. Use $(0, 0)$ and (x, y) to get $\frac{y - 0}{x - 0} = \frac{1}{2}$, $y = \frac{1}{2}x$. Use $(7, 5)$ and (x, y) to get $\frac{y - 5}{x - 7} = 2$, $y - 5 = 2(x - 7)$, $y = 2x - 9$. Solve the system of equations $y = \frac{1}{2}x$ and $y = 2x - 9$ to get $x = 6$, $y = 3$.
27. Show that opposite sides are parallel by showing that they have the same slope:
using $(3, -1)$ and $(6, 4)$, $m_1 = 5/3$; using $(6, 4)$ and $(-3, 2)$, $m_2 = 2/9$;
using $(-3, 2)$ and $(-6, -3)$, $m_3 = 5/3$; using $(-6, -3)$ and $(3, -1)$, $m_4 = 2/9$.
Opposite sides are parallel because $m_1 = m_3$ and $m_2 = m_4$.
28. The line through $(3, 1)$ and $(6, 3)$ has slope $m_1 = 2/3$, the line through $(3, 1)$ and $(2, 9)$ has slope $m_2 = -8$, the line through $(6, 3)$ and $(2, 9)$ has slope $m_3 = -3/2$. Because $m_1m_3 = -1$, the corresponding lines are perpendicular so the given points are vertices of a right triangle.

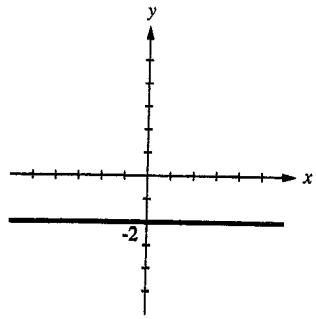
29. (a)



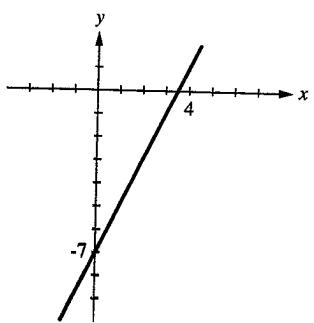
(b)



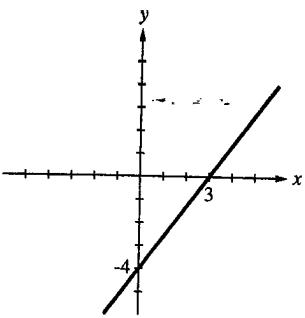
(c)



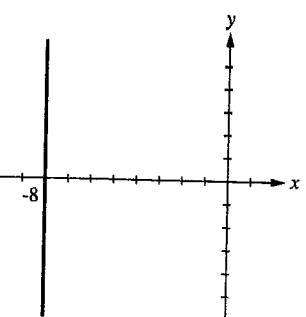
(d)



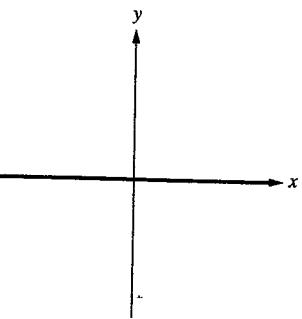
30. (a)



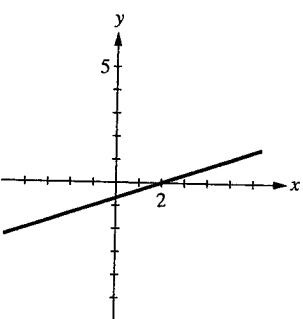
(b)



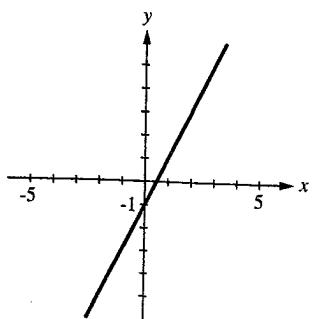
(c)



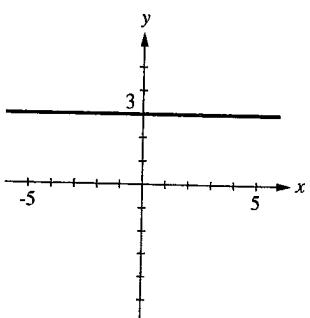
(d)



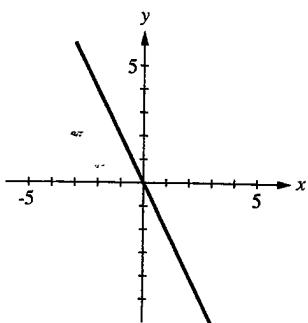
31. (a)



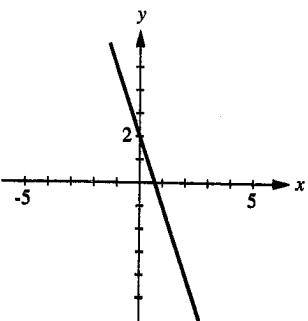
(b)



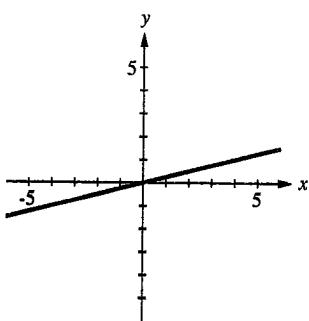
(c)



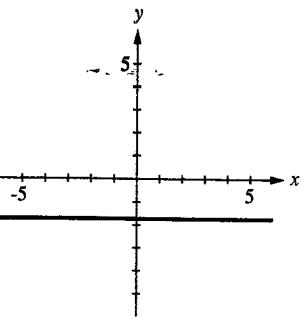
32. (a)



(b)



(c)



33. (a) $m = 3, b = 2$

(b) $m = -\frac{1}{4}, b = 3$

(c) $y = -\frac{3}{5}x + \frac{8}{5}$ so $m = -\frac{3}{5}, b = \frac{8}{5}$

(d) $m = 0, b = 1$

(e) $y = -\frac{b}{a}x + b$ so $m = -\frac{b}{a}$, y -intercept b

34. (a) $m = -4, b = 2$

(b) $y = \frac{1}{3}x - \frac{2}{3}$ so $m = \frac{1}{3}, b = -\frac{2}{3}$

(c) $y = -\frac{3}{2}x + 3$ so $m = -\frac{3}{2}, b = 3$

(d) $y = 3$ so $m = 0, b = 3$

(e) $y = -\frac{a_0}{a_1}x$ so $m = -\frac{a_0}{a_1}, b = 0$

35. (a) $m = (0 - (-3))/(2 - 0)) = 3/2$ so $y = 3x/2 - 3$

(b) $m = (-3 - 0)/(4 - 0) = -3/4$ so $y = -3x/4$

36. (a) $m = (0 - 2)/(2 - 0)) = -1$ so $y = -x + 2$

(b) $m = (2 - 0)/(3 - 0) = 2/3$ so $y = 2x/3$

37. $y = -2x + 4$

38. $y = 5x - 3$

39. The slope m of the line must equal the slope of $y = 4x - 2$, thus $m = 4$ so the equation is $y = 4x + 7$.

40. The slope of the line $3x + 2y = 5$ is $-3/2$ so the line through $(-1, 2)$ with this slope is $y - 2 = -\frac{3}{2}(x + 1)$; $y = -\frac{3}{2}x + \frac{1}{2}$.

41. The slope m of the line must be the negative reciprocal of the slope of $y = 5x + 9$, thus $m = -1/5$ and the equation is $y = -x/5 + 6$.

42. The slope of the line $x - 4y = 7$ is $1/4$ so a line perpendicular to it must have a slope of -4 ; $y + 4 = -4(x - 3)$; $y = -4x + 8$.

43. $y - 4 = \frac{-7 - 4}{1 - 2}(x - 2) = 11(x - 2)$, $y = 11x - 18$.

44. $y - 6 = \frac{1 - 6}{-2 - (-3)}(x - (-3))$, $y - 6 = -5(x + 3)$, $y = -5x - 9$.

45. The line passes through $(0, 2)$ and $(-4, 0)$, thus $m = \frac{0 - 2}{-4 - 0} = \frac{1}{2}$ so $y = \frac{1}{2}x + 2$.

46. The line passes through $(0, b)$ and $(a, 0)$, thus $m = \frac{0 - b}{a - 0} = -\frac{b}{a}$, so the equation is $y = -\frac{b}{a}x + b$.

47. $y = 1$

48. $y = -8$

49. (a) $m_1 = 4, m_2 = 4$; parallel because $m_1 = m_2$

(b) $m_1 = 2, m_2 = -1/2$; perpendicular because $m_1m_2 = -1$

(c) $m_1 = 5/3, m_2 = 5/3$; parallel because $m_1 = m_2$

(d) If $A \neq 0$ and $B \neq 0$, then $m_1 = -A/B, m_2 = B/A$ and the lines are perpendicular because $m_1m_2 = -1$. If either A or B (but not both) is zero, then the lines are perpendicular because one is horizontal and the other is vertical.

(e) $m_1 = 4, m_2 = 1/4$; neither

50. (a) $m_1 = -5, m_2 = -5$; parallel because $m_1 = m_2$

(b) $m_1 = 2, m_2 = -1/2$; perpendicular because $m_1m_2 = -1$.

(c) $m_1 = -4/5, m_2 = 5/4$; perpendicular because $m_1m_2 = -1$.

(d) If $B \neq 0$, then $m_1 = m_2 = -A/B$ and the lines are parallel because $m_1 = m_2$. If $B = 0$ (and $A \neq 0$), then the lines are parallel because they are both perpendicular to the x -axis.

(e) $m_1 = 1/2, m_2 = 2$; neither

51. $y = (-3/k)x + 4/k, k \neq 0$

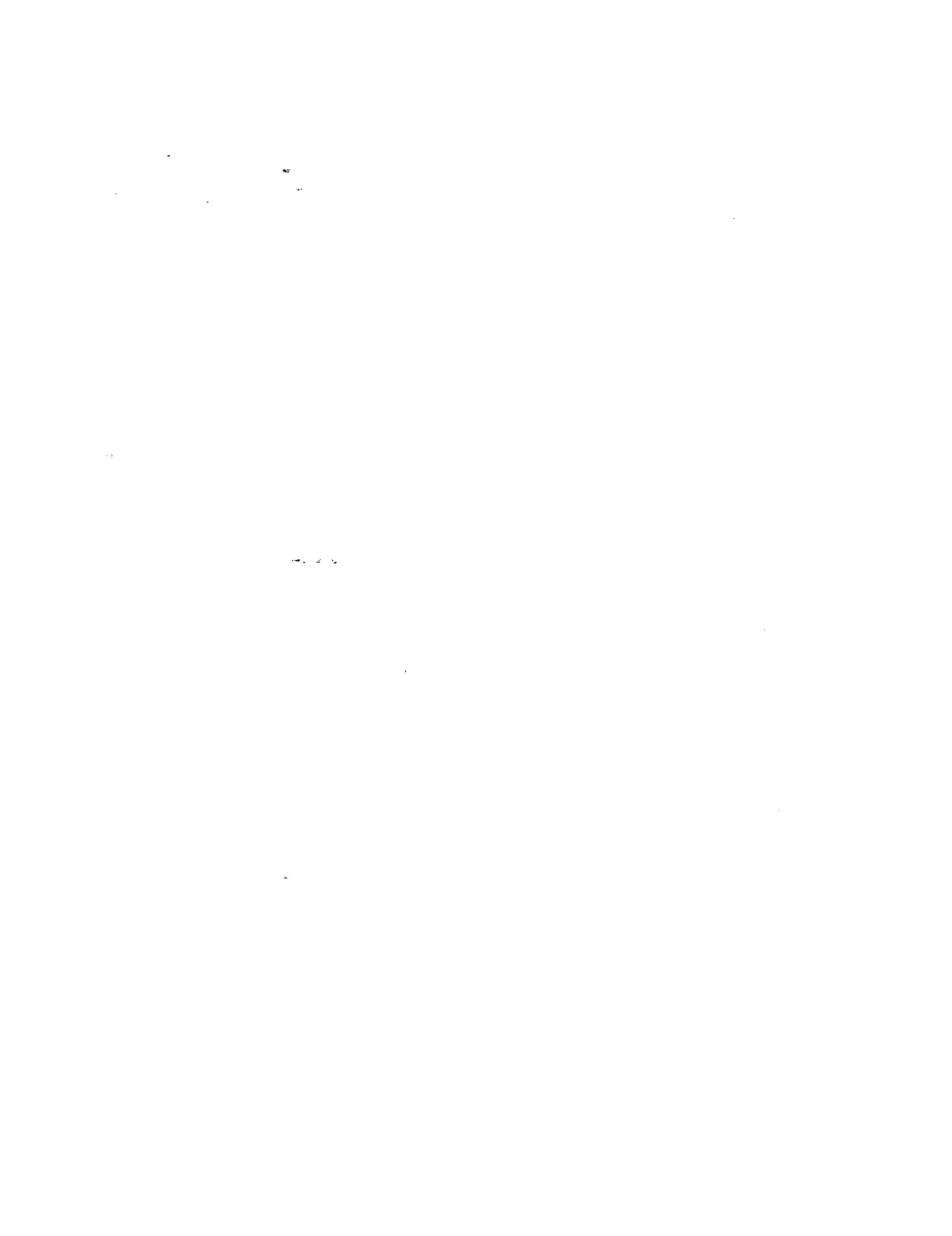
(a) $-3/k = 2, k = -3/2$

(b) $4/k = 5, k = 4/5$

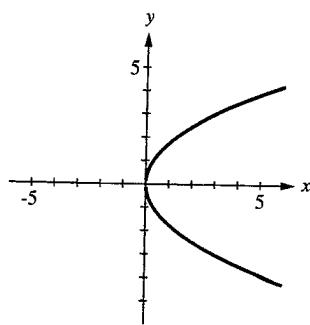
(c) $3(-2) + k(4) = 4, k = 5/2$

(d) The slope of $2x - 5y = 1$ is $2/5$ so $-3/k = 2/5, k = -15/2$.

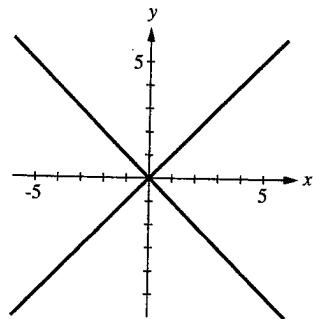
(e) The slope of $4x + 3y = 2$ is $-4/3$ so the slope of a line perpendicular to it is $3/4$; $-3/k = 3/4, k = -4$.



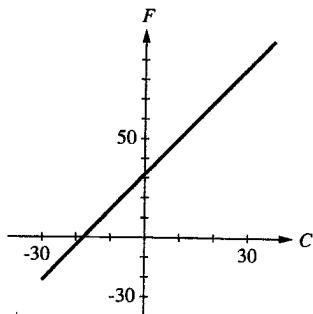
52. $y^2 = 3x$: the union of the graphs of $y = \sqrt{3x}$ and $y = -\sqrt{3x}$



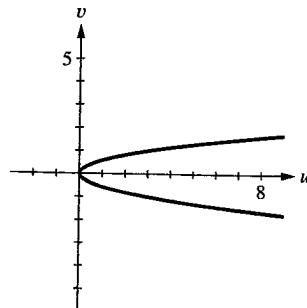
53. $(x - y)(x + y) = 0$: the union of the graphs of $x - y = 0$ and $x + y = 0$



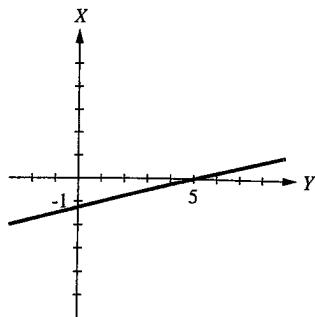
54. $F = \frac{9}{5}C + 32$



55. $u = 3v^2$



56. $Y = 4X + 5$



57. Solve $x = 5t + 2$ for t to get $t = \frac{1}{5}x - \frac{2}{5}$, so $y = \left(\frac{1}{5}x - \frac{2}{5}\right) - 3 = \frac{1}{5}x - \frac{17}{5}$, which is a line.

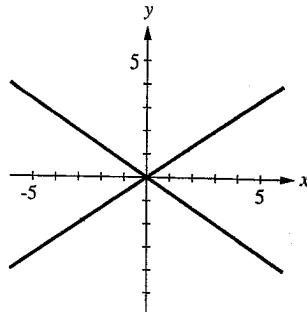
58. Solve $x = 1 + 3t^2$ for t^2 to get $t^2 = \frac{1}{3}x - \frac{1}{3}$, so $y = 2 - \left(\frac{1}{3}x - \frac{1}{3}\right) = -\frac{1}{3}x + \frac{7}{3}$, which is a line; $1 + 3t^2 \geq 1$ for all t so $x \geq 1$.

59. An equation of the line through $(1, 4)$ and $(2, 1)$ is $y = -3x + 7$. It crosses the y -axis at $y = 7$, and the x -axis at $x = 7/3$, so the area of the triangle is $\frac{1}{2}(7)(7/3) = 49/6$.

60. $(2x - 3y)(2x + 3y) = 0$, so

$$2x - 3y = 0, y = \frac{2}{3}x \text{ or } 2x + 3y = 0,$$

$y = -\frac{2}{3}x$. The graph consists of the lines $y = \pm\frac{2}{3}x$.



61. (a) yes

(b) yes

(c) no

(d) yes

(e) yes

(f) yes

(g) no