Answer.

Name: ______________________ ID Number: ____________

UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 253 — L02 FALL 2004

QUIZ #3a [04-10-27(Wed)]

Attempt all problem. Each problem: 10 marks. Total: 30 marks (best 3 out of 4 problems).

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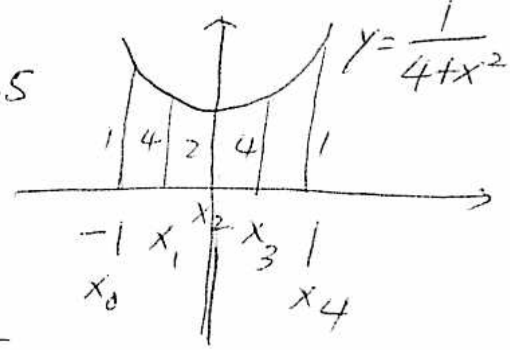
1. Using Simpson's rule and $n = 4$, evaluate $\int_{-1}^{1} \frac{dx}{4+x^2}$ up to three decimal places.

$$\int_{-1}^{1} \frac{dx}{4+x^2} = \frac{\Delta x}{3}\Big[f(-1)+4f(-0.5)+2f(0)+4f(0.5)+f(1)\Big] \qquad \Delta x = 0.5$$



$$\simeq \frac{0.5}{3}\Big[\frac{1}{5} + 4\cdot\frac{1}{4.25} + \frac{2}{4} + 4\frac{1}{4.25} + \frac{1}{5}\Big]$$

$$\simeq \frac{1}{6}\Big[.2 + .9412 + \frac{1}{2} + .9412 + .2\Big] \simeq \frac{1}{6}(2.7823)$$

$$\simeq 0.464.$$

2. Evaluate the improper integral $\int_{0}^{1/2} \frac{dx}{(1-2x)^{1/2}}$.

$\frac{1}{(1-2x)^{1/2}}$ not defined at $2x = 1$ ie $x = \frac{1}{2}$

$$\lim_{\delta \to 0^+} \int_0^{\frac{1}{2}-\delta} \frac{dx}{(1-2x)^{1/2}}$$

$$= \text{''} \quad \frac{2}{-2}(1-2x)^{1/2}\Big]_0^{\frac{1}{2}-\delta}$$

$$= \text{''} \quad -\Big[(2\delta)^{\frac{1}{2}} - 1\Big]$$

$$= \quad 1$$

3. Evaluate the improper integral $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx$

$$= \lim_{N\to\infty} \int_1^N \frac{\ln x}{x^2}\,dx \quad \text{(by parts)}$$

$$= \text{''} \quad \frac{\ln x}{-x} - \int \frac{1}{-x}\cdot\frac{1}{x}\,dx$$

$$= \text{''} \quad \frac{\ln x}{-x} - \frac{1}{x}\Big]_1^N$$

$$= \text{''} \quad \frac{\ln N}{-N} - \frac{1}{N} + \frac{\ln 1}{1} + \frac{1}{1}$$

$$= 1$$

$$\lim_{N\to\infty} \frac{\ln N}{N} = \text{''} \quad \frac{1/N}{1} = 0$$

4. Find the arc length of the curve

$$y = \frac{2}{3}(x^2-1)^{3/2}, \quad 1 \le x \le 2.$$

$$S = \int_1^2 \sqrt{1+y'^2}\,dx.$$

$$y' = \frac{\cancel{2}}{\cancel{3}}\left(\frac{\cancel{3}}{\cancel{2}}\right)(x^2-1)^{1/2}\cdot 2x$$

$$y'^2 = 4x^2(x^2-1)$$

$$1+y'^2 = 4x^4 - 4x^2 + 1$$

$$= (2x-1)^2.$$

$$\sqrt{1+y'^2} = 2x-1$$

$$= \int_1^2 2x-1\,dx$$

$$= x^2 - x\Big]_1^2$$

$$= 4-2-(1-1)$$

$$= 2.$$