

Answer

Name: _____

ID Number: _____

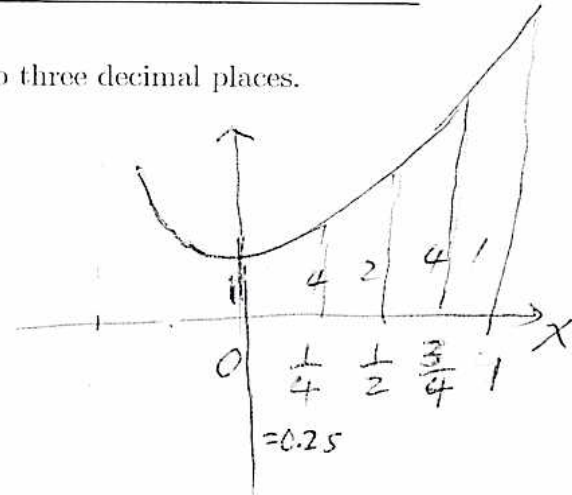
UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATHEMATICS 253 — L02 FALL 2004

QUIZ #3b [04-10-28(Thurs)]

Attempt all problem. Each problem: 10 marks. Total: 30 marks (best 3 out of 4 problems).

1. Using Simpson's rule and $n = 4$, evaluate $\int_0^1 \frac{dx}{9+x^2}$ up to three decimal places.

$$\Delta x = \frac{1}{4}$$
$$\int_0^1 \frac{dx}{9+x^2} \approx \frac{\Delta x}{3} \left[f(0) + 4f\left(\frac{1}{4}\right) + 2f\left(\frac{1}{2}\right) + 4f\left(\frac{3}{4}\right) + f(1) \right]$$



$$= \frac{1}{12} \left[\frac{1}{9} + 4\left(\frac{1}{9+\frac{1}{16}}\right) + 2\left(\frac{1}{9+\frac{1}{4}}\right) + 4\left(\frac{1}{9+\frac{9}{16}}\right) + \frac{1}{10} \right]$$

$$= \frac{1}{12} \left[\frac{1}{9} + \frac{64}{145} + \frac{8}{37} + \frac{64}{153} + \frac{1}{10} \right]$$

$$\approx \frac{1}{12} [0.1111 + 0.4414 + 0.2162 + 0.4183 + 0.1] = 0.107$$

2. Evaluate the improper integral $\int_0^1 \frac{dx}{(1-x)^{1/4}}$.

$\frac{1}{(1-x)^{1/4}}$ not defined at $x=1$

$$\lim_{\delta \rightarrow 0^+} \int_0^{1-\delta} \frac{dx}{(1-x)^{1/4}}$$

$$= \left[-\frac{4}{3} (1-x)^{3/4} \right]_0^{1-\delta}$$

$$= -\frac{4}{3} (\delta)^{3/4} + \frac{4}{3} \cdot 1$$

$$= \frac{4}{3}$$

3. Evaluate the improper integral

$$\int_1^{\infty} \frac{\ln x}{x^3} dx$$

$$= \lim_{N \rightarrow \infty} \int_1^N \frac{\ln x}{x^3} dx \quad (\text{by parts})$$

$$= \left[\frac{\ln x}{-2x^2} - \int \frac{1}{-2x^2} \cdot \frac{1}{x} dx \right]_1^N$$

$$= \left[\frac{\ln x}{-2x^2} - \frac{1}{4x^2} \right]_1^N$$

$$= \left[\frac{\ln N}{-2N^2} - \frac{1}{4N^2} + \frac{\ln 1}{2} + \frac{1}{4} \right]$$

$$= \frac{1}{4}, \quad \text{since } \lim_{N \rightarrow \infty} \frac{\ln N}{N^2} = \lim_{N \rightarrow \infty} \frac{\frac{1}{N}}{2N} = 0.$$

4. Find the arc length of the curve

$$y = \frac{1}{3}(2+x^2)^{3/2}, \quad 0 \leq x \leq 1.$$

$$s = \int_0^1 \sqrt{1+y'^2} dx \quad y' = \frac{3}{2} \cdot \frac{1}{3} (2+x^2)^{\frac{1}{2}} \cdot 2x$$

$$= x(2+x^2)^{\frac{1}{2}}$$

$$= \int_0^1 1+x^2 dx$$

$$y'^2 = x^2(2+x^2)$$

$$1+y'^2 = 1+x^2(2+x^2)$$

$$= \left[x + \frac{x^3}{3} \right]_0^1$$

$$= (1+x^2)^2$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}$$