

UNIVERSITY OF CALGARY  
DEPARTMENT OF MATHEMATICS AND STATISTICS  
MATHEMATICS 253 — L03 FALL 2004

QUIZ #5a [04-11-24(Wed)]

Attempt all problems. Each problem: 10 marks. Total: 30 marks (best 3 out of 4 problems).

1. Solve  $\frac{dy}{dx} = 0.5y - 0.05y^2$ ,  $y(0) = 9$ .

$$\frac{dy}{dx} = \frac{y}{2} - \frac{y^2}{20} = \frac{1}{20}(10y - y^2)$$

$$\frac{dy}{10y - y^2} = \frac{1}{20} dx$$

$$\frac{1}{10y - y^2} = \frac{A}{y} + \frac{B}{10 - y}$$

$$1 = A(10 - y) + By$$

$$\frac{1}{10} \int \frac{1}{y} + \frac{1}{10 - y} dy = \int \frac{1}{20} dx$$

Set

$$1 = A \cdot 10 \therefore A = \frac{1}{10}$$

$$\int \frac{1}{y} + \frac{1}{10 - y} dy = \int \frac{1}{2} dx$$

$y = 0$

$$1 = B \cdot 10 \therefore B = \frac{1}{10}$$

$y = 10$

$$\ln y - \ln(10 - y) = \frac{x}{2} + C$$

$$\ln \frac{y}{10 - y} = \frac{x}{2} + C$$

$$\frac{y}{10 - y} = Ce^{\frac{x}{2}}$$

At  $x = 0$

$$\frac{9}{1} = C$$

$$\therefore \frac{y}{10 - y} = 9e^{\frac{x}{2}}$$

2. Solve the equation  $y' + \frac{y}{x} = \ln x$

Linear eq.

$$IF e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

General Solution is

$$xy = \int x \ln x dx + C \quad (\text{by parts})$$

$$= \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx + C$$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$\therefore y = \frac{x \ln x}{2} - \frac{x}{4} + \frac{C}{x}$$

3. Solve  $y'' - 5y' + 4y = 0$ ,  $y(0) = 0$ ,  $y'(0) = 1$

$$m^2 - 5m + 4 = 0$$

$$(m-1)(m-4) = 0 \quad m = 1, 4$$

$$y = c_1 e^x + c_2 e^{4x} \quad y' = c_1 e^x + 4c_2 e^{4x}$$

At  $x=0$ ,  $0 = c_1 + c_2$

$$1 = c_1 + 4c_2$$

Subtracting

$$-1 = -3c_2 \quad \therefore c_2 = \frac{1}{3}$$

$$\therefore c_1 = -c_2 = -\frac{1}{3}$$

$$\therefore y = -\frac{1}{3}e^x + \frac{1}{3}e^{4x}$$

4. Solve  $y^{(iv)} + 2y''' + y'' = 0$ .

$$m^4 + 2m^3 + m^2 = 0$$

$$m^2(m^2 + 2m + 1) = 0$$

$$m^2(m+1)^2 = 0$$

$$\therefore m = 0, 0, -1, -1$$

General Solution is

$$y = c_1 + c_2 x + e^{-x}(c_3 + c_4 x)$$