Solution

Name: _____ ID Number: _____

UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 253 — L03 FALL 2004

QUIZ #5b [04-11-25(Thurs)]

Attempt all problems. Each problem: 10 marks. Total: 30 marks (best 3 out of 4 problems).

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1. Solve $\frac{dy}{dx} = 0.1y - 0.01y^2, \quad y(0) = 9.$

$$\frac{dy}{dx} = \frac{y}{10} - \frac{y^2}{100} = \frac{1}{100}(10y - y^2)$$

$$\frac{dy}{y(10-y)} = \frac{1}{100}\,dx$$

$$\frac{1}{y(10-y)} = \frac{A}{y} + \frac{B}{10-y}$$

$$1 = A(10-y) + By$$

Set $y=0$: $1 = A\cdot 10 \quad \therefore A = \frac{1}{10}$

$y=10$: $1 = B\cdot 10 \quad \therefore B = \frac{1}{10}$

$$\frac{1}{10}\int \frac{1}{y} + \frac{1}{10-y}\,dy = \int \frac{1}{100}\,dx$$

$$\frac{1}{10}\left(\ln y - \ln(10-y)\right) = \frac{x}{100} + C$$

$$\ln\frac{y}{10-y} = \frac{x}{10} + C_1$$

$$\frac{y}{10-y} = q\,e^{\frac{x}{10}}.$$

At $x=0$, $\frac{9}{1} = q$

$$\therefore \frac{y}{10-y} = 9e^{\frac{x}{10}}.$$

2. Solve the equation $y' + \frac{2y}{x} = e^x$

This is linear in y.

$$\text{I.F} \quad e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2.$$

General solution is

$$x^2 y = \int x^2 e^x\,dx \quad \text{(by parts)}$$

$$= x^2e^x - 2\int xe^x\,dx$$

$$= x^2e^x - 2\left[xe^x - \int e^x dx\right]$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

$$\therefore y = e^x - \frac{2e^x}{x} + \frac{2e^x}{x^2} + \frac{C}{x^2}.$$

3. Solve $3y'' - y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1$

$3m^2 - m - 2 = 0$

$(3m+2)(m-1) = 0$

$m = -\frac{2}{3}, 1.$

General Solution is

$y = c_1 e^{-\frac{2}{3}x} + c_2 e^{x}.$

$y' = -\frac{2}{3} c_1 e^{-\frac{2}{3}x} + c_2 e^{x}$

At $x=0$,

$0 = c_1 + c_2$

$1 = -\frac{2}{3} c_1 + c_2.$

$\therefore -1 = \frac{5}{3} c_1 \quad \therefore c_1 = -\frac{3}{5}, c_2 = -c_1 = \frac{3}{5}$

$\therefore y = -\frac{3}{5} e^{-\frac{2}{3}x} + \frac{3}{5} e^{x}.$

4. Solve $y^{(iv)} - 2y''' + y'' = 0.$

$m^4 - 2m^3 + m^2 = 0$

$m^2(m^2 - 2m + 1) = 0$

$m^2(m-1)^2 = 0$

$\therefore m = 0, 0, 1, 1$

General solution is

$y = c_1 + c_2 x + c_3 e^{x} + c_4 x e^{x}.$