Solution

Name: ____________________ ID Number: __________

UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 253 — L02 FALL 2004

QUIZ #2a [04-10-06(Wed)]

Attempt all problem. Each problem: 10 marks. Total: 30 marks (best 3 out of 4 problems).

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1. Evaluate

(i) $\int x(1-2x)^{2/3}\,dx$

By parts $= x.\frac{3}{5(-2)}(1-2x)^{5/3} + \frac{3}{10}\int(1-2x)^{5/3}dx$

$= \frac{-3x}{10}(1-2x)^{5/3} + \frac{3.3}{10\,8(-2)}(1-2x)^{8/3} + C$

$= -\frac{3x}{10}(1-2x)^{5/3} - \frac{9}{160}(1-2x)^{8/3} + C$

Or let $u = 1-2x$ (ie $x = \frac{1}{2}(1-u)$)

$du = -2dx$.

$\frac{1}{2}\int(1-u)u^{2/3}\frac{du}{-2} = -\frac{1}{4}\int u^{2/3} - u^{5/3}\,du$

$= -\frac{1}{4}\left[\frac{3}{5}u^{5/3} - \frac{\cdot}{8}u^{8/3}\right] + C$

$= -\frac{3}{5}\left(\frac{1}{5}(1-2x)^{5/3} - \frac{1}{8}(1-2x)^{8/3}\right) + C$

(ii) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx = \ln|e^x - e^{-x}| + C$

Or let $u = e^x - e^{-x}$.

$du = (e^x + e^{-x})dx$

$\int \frac{du}{u} = \ln|u| + c$

$= \ln|e^x - e^{-x}| + c$

2. Evaluate $\int (x+1)^2 \ln x\,dx$. By parts.

$= \frac{(x+1)^3}{3}\ln x - \int \frac{(x+1)^3}{3}\cdot\frac{1}{x}\,dx$

$=$ " $-\frac{1}{3}\int \frac{x^3+3x^2+3x+1}{x}\,dx$

$=$ " $-\frac{1}{3}\int x^2 + 3x + 3 + \frac{1}{x}\,dx$

$=$ " $-\frac{1}{3}\left[\frac{x^3}{3} + \frac{3x^2}{2} + 3x + \ln|x|\right] + C$.

3. Evaluate $\int \sin 3x \cos 4x \, dx$ by parts

$$I = \sin 3x \cdot \frac{1}{4}\sin 4x - \int 3\cos 3x \cdot \frac{1}{4}\sin 4x \, dx$$

$$= \quad '' \quad - \frac{3}{4}\left[\cos 3x\left(-\frac{1}{4}\cos 4x\right) - \int 3\sin 3x\left(-\frac{1}{4}\cos 4x\right) dx\right.$$

$$= \quad '' \quad + \frac{3}{16}\cos 3x \cos 4x + \frac{9}{16} I + C$$

$$\therefore I - \frac{9}{16} I = \quad '' \quad + \quad '' \quad + C$$

$$\frac{7}{16} I = \quad '' \quad + \quad '' \quad + C$$

$$I = \frac{16}{7}\left(\frac{1}{4}\sin 3x \sin 4x + \frac{3}{16}\cos 3x \cos 4x\right) + C_1$$

4. Evaluate $\int \frac{x^2}{\sqrt{4-x^2}} \, dx$:

Let $x = 2\sin t$

$dx = 2\cos t \, dt$.

$$\int \frac{4\sin^2 t}{2\cos t} \cdot 2\cos t \, dt$$

$$= 4\int \sin^2 t \, dt$$

$$= 4 \cdot \frac{1}{2}\int 1 - \cos 2t \, dt$$

$$= 2\left(t - \frac{1}{2}\sin 2t\right) + C$$

$$= 2\left(t - \frac{2}{2}\sin t \cos t\right) + C$$

$$= 2\left(\sin^{-1}\frac{x}{2} - \frac{x}{2}\sqrt{1-\frac{x^2}{4}}\right) + C$$

Solution

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MATHEMATICS 253 — L02 FALL 2004

QUIZ #2b [04-10-07(Thur)]

Attempt all problem. Each problem: 10 marks. Total: 30 marks (best 3 out of 4 problems).

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1. Evaluate

(i) $\int x(3x+1)^{-1/2}\,dx$ (ii) $\int \frac{x}{\sqrt{1-x^2}}\,dx$

By parts: $x\frac{2}{3}(3x+1)^{\frac{1}{2}} - \frac{2}{3}\int(3x+1)^{\frac{1}{2}}dx$

$= \frac{2x}{3}(3x+1)^{\frac{1}{2}} - \frac{2}{3}\frac{2}{3\cdot 3}(3x+1)^{\frac{3}{2}} + C$

$= \quad '' \quad - \frac{4}{27}(3x+1)^{\frac{3}{2}} + C$

or Let $u = 3x+1$ (i.e. $x = \frac{1}{3}(u-1)$)
$du = 3dx.$

$\frac{1}{3}\int(u-1)u^{-1/2}\frac{du}{3} = \frac{1}{9}\int u^{\frac{1}{2}} - u^{-\frac{1}{2}}\,du$

$= \frac{1}{9}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) + C$

$= \frac{2}{9}\left(\frac{1}{3}(3x+1)^{3/2} - (3x+1)^{1/2}\right) + C.$

(ii)

$= -\frac{1}{2}\int\frac{-2x}{\sqrt{1-x^2}}\,dx$

$= -\frac{1}{2}\cdot 2(1-x^2)^{1/2} + C$

$= -(1-x^2)^{1/2} + C.$

2. Evaluate $\int(1-x)^2\cos x\,dx.$ By parts.

$= (1-x)^2\sin x - \int 2(1-x)\sin x\,dx.$

$= \quad '' \quad -2\left[(1-x)(-\cos x) - \int -1(-\cos x)\,dx\right]$

$= \quad '' \quad +2(1-x)\cos x + 2\sin x + C.$

3. Evaluate $\int e^{-x} \sin 2x \, dx$ By parts.

$$I = -e^{-x}\sin 2x + \int e^{-x}\, 2\cos 2x \, dx$$

$$= \quad '' \quad + 2\left[-e^{-x}\cos 2x - \int -e^{-x}(-2\sin 2x)\,dx\right] + c$$

$$= \quad '' \quad - 2e^{-x}\cos 2x - 4\int e^{-x}\sin 2x\,dx + c$$

$$= \quad '' \quad - \quad '' \quad - 4I + c$$

$$\therefore I + 4I = \quad '' \quad - \quad '' \quad + c$$

$$I = \frac{1}{5}\left(-e^{-x}\sin 2x - 2e^{-x}\cos 2x\right) + c.$$

4. Evaluate $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$: Let $x = 3\sin t$. $(9 - x^2 = 9 - 9\sin^2 t = 9\cos^2 t)$

$dx = 3\cos t \, dt$

$$\int \frac{9\sin^2 t}{3\cos t} \cdot 3\cos t \, dt$$

$$= 9\int \sin^2 t \, dt$$

$$= 9 \cdot \frac{1}{2}\int 1 - \cos 2t \, dt$$

$$= \frac{9}{2}\left[t - \frac{1}{2}\sin 2t\right] + c$$

$$= \frac{9}{2}\left(t - \frac{2}{2}\sin t\cos t\right) + c$$

$$= \frac{9}{2}\left(\sin^{-1}\frac{x}{3} - \frac{x}{3}\sqrt{1 - \frac{x^2}{9}}\right) + c.$$