

UNIVERSITY OF CALGARY  
DEPARTMENT OF MATHEMATICS AND STATISTICS  
MATHEMATICS 253 — L02      FALL 2004  
MIDTERM REVIEW

Midterm exam:      Oct 20, 2004

1. Find the inverse functions of the following functions and their domain and range

(a)  $f(x) = \frac{1-x}{3x-2}$ .

(b)  $f(x) = \sin(3x+2)$ .

2. Evaluate:

(a)  $\int \sin^{-1} 2x \, dx$ .

(b)  $\int_0^{\pi/4} x \sin 2x \, dx$ .

(c)  $\int_1^2 x(\ln x)^2 \, dx$ .

(d)  $\int \frac{x}{2x^2 - x + 2} \, dx$ .

(e)  $\int \frac{x+1}{2x^2 - x - 1} \, dx$ .

(f)  $\int \frac{x^2}{\sqrt{4x^2 + 9}} \, dx$ .

(g)  $\int_0^1 x^2 e^{-2x} \, dx$ .

3. Evaluate the improper integrals

(a)  $\int_0^2 \frac{dx}{x^{1/2}}$ .

(b)  $\int_1^{\infty} \ln x \, dx$ .

(c)  $\int_1^{\infty} \frac{\ln x}{x^2} \, dx$ .

(d)  $\int_0^1 x^2 \ln x \, dx$ .

4. Find  $F'(x)$  and  $F''(x)$ , if

(a)  $F(x) = \int_{-x}^{2x} \sin 6x \, dx$

(b)  $F(x) = \int_{2x}^{x^2} \ln(x+1) \, dx.$

5. Find the arclength of the curve over the given interval

(a)  $y = e^x + e^{-x}$  from  $x = 0$  to  $x = 1$ .

(b)  $x = \frac{y^4}{8} + \frac{1}{4y^2}$  from  $y = 1$  to  $y = 4$ .

(c)  $y = \cos 2t$ ,  $x = \sin 2t$  from  $t = 0$  to  $t = \frac{\pi}{4}$ .

**HINT:**  $\int_0^{\pi/4} \sqrt{x'^2 + y'^2} \, dt.$