Midterm exam: *Oct 20, 2004*

1. Find the inverse functions of the following functions and their domain and range

   (a) $f(x) = \dfrac{1-x}{3x-2}$.

   (b) $f(x) = \sin(3x+2)$.

2. Evaluate:

   (a) $\displaystyle\int \sin^{-1} 2x\, dx$.

   (b) $\displaystyle\int_0^{\pi/4} x\sin 2x\, dx$.

   (c) $\displaystyle\int_1^2 x(\ln x)^2\, dx$.

   (d) $\displaystyle\int \dfrac{x}{2x^2 - x + 2}\, dx$.

   (e) $\displaystyle\int \dfrac{x+1}{2x^2 - x - 1}\, dx$.

   (f) $\displaystyle\int \dfrac{x^2}{\sqrt{4x^2 + 9}}\, dx$.

   (g) $\displaystyle\int_0^1 x^2 e^{-2x}\, dx$.

3. Evaluate the improper integrals

   (a) $\displaystyle\int_0^2 \dfrac{dx}{x^{1/2}}$.

   (b) $\displaystyle\int_1^\infty \ln x\, dx$.

   (c) $\displaystyle\int_1^\infty \dfrac{\ln x}{x^2}\, dx$.

   (d) $\displaystyle\int_0^1 x^2 \ln x\, dx$.

4. Find $F'(x)$ and $F''(x)$, if

   (a) $F(x) = \int_{-x}^{2x} \sin 6x \, dx$

   (b) $F(x) = \int_{2x}^{x^2} \ln(x+1) \, dx.$

5. Find the arclength of the curve over the given interval

   (a) $y = e^x + e^{-x}$ from $x = 0$ to $x = 1$.

   (b) $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}$ from $y = 1$ to $y = 4$.

   (c) $y = \cos 2t$, $x = \sin 2t$ from $t = 0$ to $t = \dfrac{\pi}{4}$.

   HINT: $\int_0^{\pi/4} \sqrt{x'^2 + y'^2} \, dt.$