

Answer

UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATHEMATICS 253 — L02 FALL 2004
MIDTERM EXAM [04-10-20(Wed)]

Time: 50 minutes. PLEASE write your Name on the very last page.

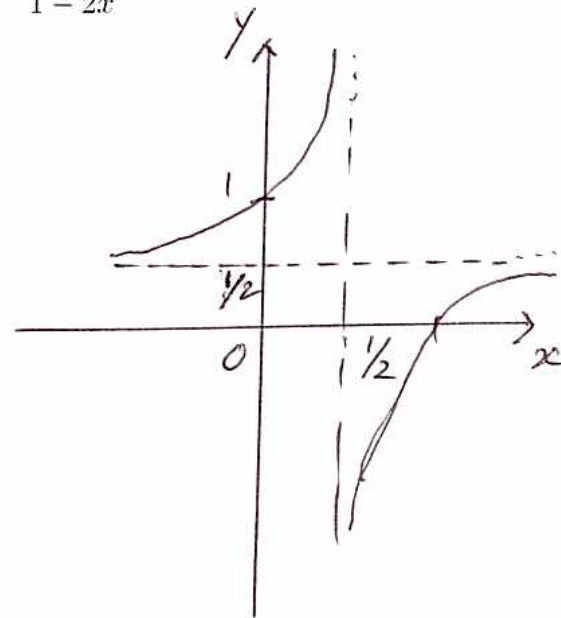
Student ID: _____

1. (a) Find the inverse function of
and find the domain of the inverse function.:

$$f(x) = \frac{1-x}{1-2x}$$

$$x = \frac{1-y}{1-2y}$$
$$x(1-2y) = 1-y$$
$$y(1-2x) = 1-x$$
$$y = \frac{1-x}{1-2x}$$

$$\text{Domain: } x \neq \frac{1}{2}$$



- (b) Find the domain and range of the inverse function of

$$f(x) = 2 \sin 2x$$

$$x = 2 \sin 2y$$
$$\frac{x}{2} = \sin 2y$$
$$2y = \sin^{-1} \frac{x}{2}$$
$$y = \frac{1}{2} \sin^{-1} \frac{x}{2}$$

$$\text{Domain: } -1 \leq \frac{x}{2} \leq 1 \quad \text{ie } -2 \leq x \leq 2$$

$$\text{Range: } \frac{1}{2} \left(-\frac{\pi}{2}\right) \leq y \leq \frac{1}{2} \left(\frac{\pi}{2}\right)$$

$$\therefore -\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$$

2. Evaluate $\int_1^e x^2 \ln x \, dx$ by parts

$$= \left. \frac{x^3}{3} \ln x \right|_1^e - \int_1^e \frac{x^3}{3} \cdot \frac{1}{x} \, dx$$

$$= \left. \frac{x^3}{3} \ln x - \frac{x^3}{3 \cdot 3} \right|_1^e$$

$$= \frac{e^3}{3} \ln e - \frac{e^3}{9} - \left(\frac{1}{3} \ln 1 - \frac{1}{9} \right)$$

$$= \frac{e^3}{3} - \frac{e^3}{9} + \frac{1}{9}$$

$$= \frac{2e^3}{9} + \frac{1}{9}$$

3. Evaluate $\int \frac{x}{x^2 + 2x + 5} dx$

$$= \int \frac{x}{(x+1)^2 - 1 + 5} dx$$

$$= \int \frac{x}{(x+1)^2 + 4} dx$$

Let $u = x+1$ ($x = u-1$)
 $du = dx$

$$= \int \frac{u}{u^2 + 4} - \frac{1}{u^2 + 4} du$$

$$= \frac{1}{2} \ln(u^2 + 4) - \frac{1}{2} \tan^{-1} \frac{u}{2} + C$$

$$= \frac{1}{2} \ln(x^2 + 2x + 5) - \frac{1}{2} \tan^{-1} \frac{x+1}{2} + C.$$

4. Evaluate the improper integral $\int_1^{\infty} \frac{dx}{x^{2/3}}$.

$$= \lim_{N \rightarrow \infty} \int_1^N \frac{dx}{x^{2/3}}$$

$$= \text{''} \quad \left[3x^{1/3} \right]_1^N$$

$$= \text{''} \quad 3N^{1/3} - 3.$$

$$= \infty. \quad (\text{div.})$$

5. If $F(x) = \int_{2x}^{x^2} \sin t^2 dt$,
find

(a) $F'(x)$

$$\begin{aligned} F'(x) &= \sin x^4 \cdot 2x - \sin 4x^2 \cdot 2 \\ &= 2x \sin x^4 - 2 \sin 4x^2 \end{aligned}$$

(b) $F''(x)$.

$$F'' = 2 \sin x^4 - 8x^4 \cos x^4 - 16x \cos 4x^2.$$

Name: _____