

Final Review (Solution)

$$\begin{aligned}
 & \text{I (i) } \int \frac{x}{x^2 - x + 2} dx \\
 & \quad x^2 - x + 2 = (x - \frac{1}{2})^2 - \frac{1}{4} + 2 = \\
 & \quad = (x - \frac{1}{2})^2 + \frac{7}{4} \\
 & = \int \frac{u + \frac{1}{2}}{u^2 + \frac{7}{4}} du \quad \text{Let } u = x - \frac{1}{2} \\
 & \quad du = dx \\
 & = \int \frac{u}{u^2 + \frac{7}{4}} + \frac{\frac{1}{2}}{u^2 + \frac{7}{4}} du \\
 & = \frac{1}{2} \ln(u^2 + \frac{7}{4}) + \frac{1}{2} \cdot \frac{1}{\sqrt{7}} \tan^{-1} \frac{u}{\frac{\sqrt{7}}{2}} + C \\
 & = \frac{1}{2} \ln(x^2 - x + 2) + \frac{1}{\sqrt{7}} \tan^{-1} \frac{2(x - \frac{1}{2})}{\sqrt{7}} + C.
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ii) } \int \frac{\sqrt{x^2 - 1}}{x} dx \quad \text{Let } x = \sec \theta \\
 & \quad dx = \sec \theta \tan \theta d\theta. \quad \frac{x}{\theta} \sqrt{x^2 - 1} \\
 & = \int \frac{\tan \theta}{\sec \theta} \sec \theta \tan \theta d\theta \\
 & = \int \tan^2 \theta d\theta = \int \sec^2 \theta - 1 d\theta = \tan \theta - \theta + C \\
 & = \sqrt{x^2 - 1} - \sec^{-1} x + C.
 \end{aligned}$$

$$\begin{aligned}
 & \text{(iii) } \int \sqrt{4x^2 + 8x + 25} dx \\
 & = \int \sqrt{4(x+1)^2 + 21} dx \\
 & = \int \sqrt{21} \sec \theta \cdot \frac{\sqrt{21}}{2} \sec^2 \theta d\theta \quad \text{Let } x+1 = \frac{\sqrt{21}}{2} \tan \theta \\
 & \quad dx = \frac{\sqrt{21}}{2} \sec^2 \theta d\theta \\
 & = \frac{\sqrt{21}}{2} \int \sec^3 \theta d\theta. \quad (\text{use formula})
 \end{aligned}$$

$$\begin{aligned}
 & \text{Now } I = \int \sec^3 \theta d\theta = \int \sec \theta \sec^2 \theta d\theta = \sec \theta \cdot \tan \theta - \int \sec \theta \tan \theta \tan \theta d\theta \\
 & = \sec \theta \tan \theta - \int \sec \theta (\sec^2 \theta - 1) d\theta \\
 & = " - I + \int \sec \theta d\theta \\
 & \therefore 2I = " + \ln |\sec \theta + \tan \theta| \quad \text{Diagram: A right triangle with hypotenuse } \sqrt{21}, \text{ vertical leg } 2(x+1), \text{ and horizontal leg } 2(x+1).
 \end{aligned}$$

$$1(iv). \int \frac{\sqrt{9+4x^2}}{x} dx \quad \text{Let } x = \frac{3}{2} \tan \theta \\ dx = \frac{3}{2} \sec^2 \theta d\theta$$

$$= \int \frac{3 \sec \theta}{\frac{3}{2} \tan \theta} \cdot \frac{3}{2} \sec^2 \theta d\theta.$$

$$= 3 \int \frac{\sec \theta (1 + \tan^2 \theta)}{\tan \theta} d\theta$$

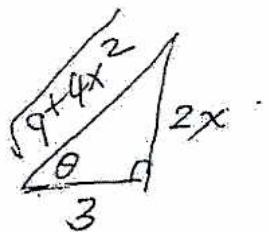
$$= 3 \int \frac{\sec \theta}{\tan \theta} + \sec \theta \tan \theta d\theta$$

$$= 3 \int \frac{1}{\sin \theta} + " d\theta$$

$$= 3 \int \csc \theta + " d\theta$$

$$= 3 \left[ -\ln |\csc \theta + \cot \theta| + \sec \theta \right] + C$$

$$= 3 \left[ -\ln \left| \frac{\sqrt{9+4x^2}}{2x} + \frac{3}{2x} \right| + \frac{\sqrt{9+4x^2}}{9} \right] + C$$



$$(iv) \int \frac{\sqrt{9+4x^2}}{x} dx \quad \text{Let } x = \frac{3}{2} \tan \theta. \quad \text{etc.}$$

$$(v) \int_0^{1/2} x^2 \sin^{-1} x dx \quad (\text{by parts})$$

$$= \frac{x^3}{2} \cdot \sin^{-1} x - \int \frac{x^3}{2} \frac{dx}{\sqrt{1-x^2}}. \quad \text{Let } u = 1-x^2 \text{ or } u = \sin \theta$$

$$= " \quad - \frac{1}{2} \int \frac{x^2 - x dx}{\sqrt{1-x^2}} \quad du = -2x dx \\ \downarrow$$

$$= " \quad - \frac{1}{2} \int (1-u) \frac{du}{-2\sqrt{u}} \quad \frac{du}{-2} = x dx.$$

$$= " \quad + \frac{1}{4} \int \frac{1-u}{u^{1/2}} du$$

$$= " \quad + \frac{1}{4} \int \frac{1}{u^{1/2}} - \frac{u}{u^{1/2}} du$$

$$= " \quad + \frac{1}{4} \left[ 2u^{1/2} - \frac{2}{3}u^{3/2} \right] + C. \quad (\text{subst. } u = 1-x^2)$$

(vi) Improper integral

$$\lim_{N \rightarrow \infty} \int_2^N \frac{\ln x}{x} dx \quad u = \ln x \quad du = \frac{1}{x} dx$$

$$= " \int u du$$

$$= " \quad \frac{u^2}{2}$$

$$= " \quad \frac{(\ln x)^2}{2} \Big|_2^N$$

$$= " \quad \frac{(\ln N)^2}{2} - \frac{(\ln 2)^2}{2} = \infty \quad (\text{div.})$$

$$(vii) \lim_{N \rightarrow \infty} \int_1^N \frac{\ln x}{x^3} dx \quad (\text{by parts})$$

$$= " \quad \left[ \frac{1}{2x^2} \ln x \right]_1^N - \int_1^N \frac{1}{-2x^2} \frac{1}{x} dx$$

$$= " \quad \frac{\ln N}{-2x^2} \Big|_1^N - \frac{1}{4x^2} \Big|_1^N + \frac{1}{2} = \frac{1}{2} \quad \begin{aligned} & \text{l'Hopital's rule} \\ & \lim_{N \rightarrow \infty} \frac{\ln N}{N^2} = \lim_{N \rightarrow \infty} \frac{1}{2N} \end{aligned}$$

$$(viii) \int_0^{\frac{\pi}{4}} x \tan^{-1} 2x \, dx \quad (\text{by parts})$$

$$= \frac{x^2}{2} \tan^{-1} 2x - \int x \cdot \frac{2}{1+4x^2} \, dx.$$

$$= " - \frac{1}{4} \ln(1+4x^2) \Big|_0^{\frac{\pi}{4}}$$

$$= \frac{\pi^2}{32} \tan^{-1} \frac{\pi}{2} - \frac{1}{4} \ln\left(1 + \frac{\pi^2}{4}\right) = 0$$

$$= \frac{\pi^2}{32} - \frac{1}{4} \ln\left(1 + \frac{\pi^2}{4}\right).$$

$$(ix) \text{ Improper } \lim_{N \rightarrow \infty} \int_0^N x^3 e^{-x} \, dx \quad (\text{by parts})$$

$$= " - x^3 e^{-x} + \int 3x^2 e^{-x} \, dx$$

$$= " " + 3 \left[ -x^2 e^{-x} + \int 2x e^{-x} \, dx \right]$$

$$= " " + 3 \left[ -x e^{-x} + 2x e^{-x} + 2 \int e^{-x} \, dx \right]$$

$$= " - e^{-x} (x^3 + 3x^2 + 6x + 6) \Big|_0^N$$

$$= + 6$$

*l'Hopital's rule*

$$\lim_{N \rightarrow \infty} e^{-x} x^3 = \lim_{N \rightarrow \infty} \frac{x^3}{e^x}$$

$$= " \frac{3x^2}{e^x}$$

$$= " \frac{6x}{e^x}$$

$$= " \frac{6}{e^x} = 0.$$

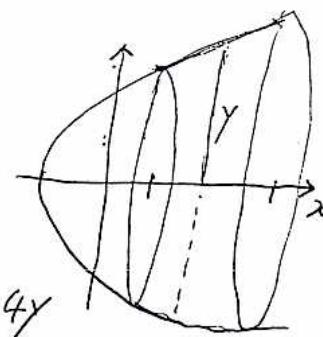
$$2 \text{ pg 559, 12. } x = 1 + 2y^2, \quad 1 \leq y \leq 2.$$

$$\text{Surface Area} = \int_1^2 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$= 2\pi \int_1^2 y \sqrt{1+16y^2} dy, \quad \frac{dx}{dy} = 4y$$

$$= \frac{2\pi}{32} \int_1^2 32y(1+16y^2)^{\frac{1}{2}} dy, \quad \left(\frac{dx}{dy}\right)^2 = 16y^2$$

$$= \frac{\pi}{16} \cdot \frac{2}{3} (1+16y^2)^{\frac{3}{2}} \Big|_1^2 = \frac{\pi}{24} \left( 65^{\frac{3}{2}} - 17^{\frac{3}{2}} \right).$$



pg 559, 16

$$\text{Surface Area} = \int_{-a}^a 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$= 2\pi \int_{-a}^a a \cosh \frac{y}{a} \cdot \cosh \frac{y}{a} dy, \quad \frac{dx}{dy} = a \cdot \frac{1}{a} \sinh \frac{y}{a}$$

$$= 2\pi a \int_{-a}^a \frac{1}{2} \left( 1 + \cosh \frac{2y}{a} \right) dy, \quad \left( \frac{dx}{dy} \right)^2 = \sinh^2 \frac{y}{a}$$

$$= \pi a \left[ y + \frac{a}{2} \sinh \frac{2y}{a} \right]_{-a}^a, \quad 1+x'^2 = 1+\sinh^2 \frac{y}{a}$$

$$= \frac{1}{2} \left( 1 + \cosh \frac{2y}{a} \right)$$

$$= \pi a \left( a + \frac{a}{2} \sinh 2 - (-a + \frac{a}{2} \sinh(-2)) \right)$$

$$= \pi a (2a + a \sinh 2) = \pi a^2 (2 + \sinh 2).$$

$$4. (i) \quad y' - y = 2x, \quad y(1) = 1$$

$$IF = e^{-\int 1 dx} = e^{-x}.$$

$$\therefore e^{-x} y = 2 \int x e^{-x} dx = 2 \left[ -x e^{-x} + \int e^{-x} dx \right]$$

$$= 2(-x e^{-x} - e^{-x}) + C.$$

$$\text{Using } y(1) = 1,$$

$$\begin{aligned} \therefore y &= -2x - 2 + ce^x \\ &= -4 + ce^x \quad \therefore c = 5e^{-1} \end{aligned}$$

$$\therefore y = -2x - 2 + 5e^{x-1}$$

$$(ii) \quad y' - \frac{y}{x} = x, \quad y(1) = 1.$$

$$IF = e^{-\int \frac{1}{x} dx} = e^{-\ln x} = x^{-1}.$$

$$\therefore x^{-1}y = \int x \cdot x^{-1} dx = \int dx = x + c$$

$$\therefore y = x^2 + cx$$

Using initial condition  $y(1) = 1$ , we get

$$1 = 1 + c \quad \therefore c = 0.$$

$$\text{Hence } y = x^2.$$

{ (iii). See (iv) below (except  $v(0) = 0$ ). }

{ (iv). Let  $x' = v$

$$\text{Then } v' = 32 - \frac{v}{2}. \quad v(0) = 10.$$

$$v' = \frac{64-v}{2} \quad (\text{Separable})$$

$$\int_{10}^v \frac{dv}{64-v} = \int_0^t \frac{dt}{2}.$$

$$-\ln|64-v| \Big|_{10}^v = \frac{t}{2} \Big|_0^t$$

$$-\ln|64-v| + \ln 54 = \frac{t}{2}.$$

$$\ln \frac{54}{164-v} = \frac{t}{2} \quad t/2.$$

$$54 = (64-v)e^{\frac{t}{2}}$$

$$54e^{-\frac{t}{2}} = 64-v$$

$$v = 64 - 54e^{-\frac{t}{2}}$$

$$dy/dt = 64 - 54e^{-\frac{t}{2}}$$

$$\therefore x \Big|_0^x = \int_0^t 64 - 54e^{-t/2} dt$$

$$\therefore x = 64t + 108e^{-t/2} \Big|_0^t$$

$$= 64t + 108e^{-t/2} - 108.$$

$$(vi) \frac{dy}{dt} + 2y = e^{3x}, y(0) = 1$$

$$I.F. = e^{\int 2 dx} = e^{2x}$$

$$\therefore e^{2x} y = \int e^{2x} e^{3x} dx = \frac{1}{5} e^{5x} + C$$

$$\therefore y = \frac{1}{5} e^{3x} + C e^{-2x}.$$

$$\text{From } y(0) = 1, 1 = \frac{1}{5} + C \therefore C = \frac{4}{5}$$

$$\therefore y = \frac{1}{5} e^{3x} + \frac{4}{5} e^{-2x}$$

$$(vii) y' + \frac{y}{2} = \frac{1}{2} e^{-x} \quad \text{similar to (v). I.F.} = e^{-\int \frac{1}{2} dx} = e^{-\frac{x}{2}}.$$

$$(vii) y'' + 2y' + y = 0, y(0) = 0, y'(0) = 1$$

$$\text{Ch. eq. } m^2 + 2m + 1 = 0 \quad \therefore (m+1)^2 = 0, \therefore m = -1, -1$$

$$\therefore y = e^{-x} (c_1 + c_2 x) \quad + y' = e^{-x} [-(c_1 + c_2 x) + c_2]$$

$$= e^{-x} (-c_1 + c_2 - c_2 x)$$

Hence

$$0 = e^0 (c_1) \therefore c_1 = 0$$

$$1 = (c_2) \therefore c_2 = 1 \leftarrow$$

$$\therefore \underline{y = e^{-x} x}$$

$$(viii) \int_0^{\pi/4} x \tan^{-1} 2x \, dx \quad (\text{by parts})$$

$$= \frac{x^2}{2} \tan^{-1} 2x - \int x \cdot \frac{2}{1+4x^2} \, dx.$$

$$\therefore \quad " \quad - \frac{1}{4} \ln(1+4x^2) \Big|_0^{\pi/4}$$

$$\therefore \frac{\pi^2}{32} \tan^{-1} \frac{\pi}{2} - \frac{1}{4} \ln\left(1 + \frac{\pi^2}{4}\right) = 0$$

$$\therefore \frac{\pi^2}{32} = -\frac{1}{4} \ln\left(1 + \frac{\pi^2}{4}\right).$$

$$(ix) \text{ improper } \lim_{N \rightarrow \infty} \int_0^N x^3 e^{-x} \, dx \quad (\text{by parts})$$

$$= " \quad -x^3 e^{-x} + \int 3x^2 e^{-x} \, dx$$

$$= " \quad " + 3 \left[ -x^2 e^{-x} + \int 2x e^{-x} \, dx \right]$$

$$= " \quad " + 3 \left[ -x e^{-x} + 2x e^{-x} + 2 \int e^{-x} \, dx \right]$$

$$= " \quad -e^{-x} (x^3 + 3x^2 + 6x + 6) \Big|_0^N \quad " .$$

$$= + 6$$

l'Hopital's rule

$$\lim_{N \rightarrow \infty} e^{-x} x^3 = \lim_{N \rightarrow \infty} \frac{x^3}{e^x}$$

$$= " \quad \frac{3x^2}{e^x}$$

$$= " \quad \frac{6x}{e^x}$$

$$= " \quad \frac{6}{e^x} = 0 .$$

3 pg. 645/6

$$\frac{dx}{dt} = 1-t+x(1-t) = (1+x)(1-t)$$

$$\int \frac{dx}{1+x} = \int (1-t)dt \quad \text{Separable.}$$

$$\ln \frac{1+x}{c} = t - \frac{t^2}{2}$$
$$1+x = ce^{t - \frac{t^2}{2}}$$

pg 645/8

$$x^2 y' - y = 2x^3 e^{-\frac{1}{x}}$$

$$y' - \frac{y}{x^2} = 2x e^{-\frac{1}{x}} \quad \text{Linear}$$

$$IF = e^{\int \frac{1}{x^2} dx} = e^{\frac{1}{x}}$$

∴ General solution is

$$e^{\frac{1}{x}} y = \int 2x e^{-\frac{1}{x}} e^{\frac{1}{x}} dx$$
$$= \int 2x dx$$
$$= x^2 + C$$
$$\therefore y = (x^2 + C) e^{-\frac{1}{x}}.$$

5 pg 1147 / 22.

$$y'' - 2y' + 5y = 0, \quad y(\pi) = 0, \quad y'(\pi) = 2.$$

Let  $y = e^{mx}$ . Then

$$m^2 - 2m + 5 = 0 \quad \therefore m = \frac{2 \pm \sqrt{4 - 4 \cdot 5 \cdot 1}}{2} = \frac{2 \pm \sqrt{-16}}{2}$$

∴ The general solution is  $= \frac{2 \pm 4i}{2} = 1 \pm 2i$

$$y(x) = e^x (c_1 \cos 2x + c_2 \sin 2x)$$

$$y'(x) = e^x (c_1 \cos 2x + c_2 \sin 2x - 2c_1 \sin 2x + 2c_2 \cos 2x)$$

$$\text{At } x = \pi, \quad y(\pi) = e^\pi (c_1 \cos 2\pi + c_2 \sin 2\pi) = e^\pi c_1 \cdot 1 = 0 \quad \therefore c_1 = 0$$

$$y'(\pi) = e^\pi (c_1 + 2c_2) = 2 \quad \therefore c_2 = e^{-\pi}$$

$$\therefore y(x) = e^x \cdot e^{-\pi} \sin 2x = e^{x-\pi} \sin 2x.$$

p 1147 / 24

$$y'' + 12y' + 36y = 0, \quad y(1) = 0, \quad y'(1) = 1.$$

$$\text{Ch. eq. } m^2 + 12m + 36 = 0$$

$$(m+6)^2 = 0 \quad \therefore m = -6, -6$$

$$\therefore y(x) = e^{-6x} (c_1 + c_2 x)$$

$$y'(x) = e^{-6x} [-6(c_1 + c_2 x) + c_2] = e^{-6x} (-6c_1 - 6c_2 x + c_2)$$

$$\text{At } x = 1, \quad y(1) = e^{-6} (c_1 + c_2) = 0 \quad \therefore c_1 + c_2 = 0 \quad \text{--- (1)}$$

$$y'(1) = e^{-6} (-6c_1 - 6c_2 + c_2) = 1 \quad \therefore -6c_1 + 5c_2 = e^6 \quad \text{--- (2)}$$

$$5 \times (1) \quad 5c_1 + 5c_2 = 0 \quad \text{--- (3)}$$

$$(2) - (3) \quad -c_1 = e^6 \quad \therefore c_1 = -e^6 \\ c_2 = -c_1 = e^6.$$

$$\therefore y(x) = e^{-6x} (-e^6 + e^6 x) = e^{-6x} e^6 (-1 + x)$$

$$= e^{-6(x-1)} (-1 + x).$$

P. 1147/28.  $y'' + 100y = 0, \quad y(0) = 2, \quad y(\pi) = 5$

 $m^2 + 100 = 0$ 
 $m^2 = -100 \quad \therefore m = \pm 10i$

$\therefore y = c_1 \cos 10x + c_2 \sin 10x.$

$y(0) = c_1 \cdot 1 = 2.$

$y(\pi) = c_1 \cos(10\pi) + c_2 \sin(10\pi) = c_1 \cdot 1 = 5.$

$\therefore c_1$  is inconsistent (i.e. no solution)

P. 1147/30.  $y'' - 6y' + 9y = 0, \quad y(0) = 1, \quad y(1) = 0$

$m^2 - 6m + 9 = 0$

$(m-3)^2 = 0 \quad \therefore m = 3, 3$

$y(x) = e^{3x} (c_1 + c_2 x).$

$y(0) = c_1 = 1$

$y(1) = e^3 (c_1 + c_2) = 0 \quad \therefore c_1 + c_2 = 0$

$\therefore c_2 = -c_1 = -1$

$\therefore y(x) = e^{3x} (1 - x).$

Pg 1154/7 Non-homo. D.E.

$y'' + y = e^x + x^3, \quad y(0) = 2, \quad y'(0) = 0.$

Ch. eq.  $m^2 + 1 = 0 \quad \therefore m = \pm i$

$y_c(x) = c_1 \cos x + c_2 \sin x$

UC sets  $\{e^x\}, \{x^3, x^2, x, 1\}$

Try  $y_p(x) = Ae^x + Bx^3 + Cx^2 + Dx + E.$

$y_p' = Ae^x + 3Bx^2 + 2Cx + D$

$y_p'' = Ae^x + 6Bx + 2C$

17 (ctd.)

$$\therefore y_p'' + y_p = 2Ae^x + Bx^3 + Cx^2 + (D+6B)x + E + 2C \\ = e^x + x^3.$$

Equating coeff of like terms:

$$\begin{array}{ll} e^x: & 2A = 1 \\ x^3: & B = 1 \\ x^2: & C = 0 \\ x: & D+6B = 0 \\ 1: & E+2C = 0 \end{array}$$

$$\therefore A = \frac{1}{2}, B = 1, C = 0, D = -6B = -6, E = -2C = 0$$

$$\therefore y_p(x) = \frac{1}{2}e^x + x^3 - 6x$$

$$\therefore y(x) = y_c + y_p = c_1 \cos x + c_2 \sin x + \frac{1}{2}e^x + x^3 - 6x.$$

Using initial conditions to find  $c_1$  &  $c_2$ :

$$y'(x) = -c_1 \sin x + c_2 \cos x + \frac{1}{2}e^x + 3x^2 - 6,$$

$$\text{At } x=0, \quad y(0) = c_1 + \frac{1}{2} = 2 \quad \therefore c_1 = 2 - \frac{1}{2} = \frac{3}{2}$$

$$y'(0) = c_2 + \frac{1}{2} - 6 = 0. \quad c_2 = 6 - \frac{1}{2} = \frac{11}{2}.$$

$$\therefore y(x) = \frac{3}{2} \cos x + \frac{11}{2} \sin x + \frac{1}{2}e^x + x^3 - 6x.$$

$$\text{P1154/10. } y'' + y' - 2y = x + \sin 2x, \quad y(0) = 1, y'(0) = 0$$

$$\text{To find } y_c: \quad m^2 + m - 2 = 0 \\ (m-1)(m+2) = 0 \quad \therefore m = 1, -2.$$

$$\therefore y_c(x) = c_1 e^x + c_2 e^{-2x}$$

U.C. sets are  $\{x, 1\}$  and  $\{\sin 2x, \cos 2x\}$

$$y_p(x) = Ax + B + C \sin 2x + D \cos 2x.$$

$$Y_p'(x) = A + 2C \cos 2x - 2D \sin 2x$$

$$Y_p''(x) = -4C \sin 2x - 4D \cos 2x$$

$$\begin{aligned} \therefore Y_p'' + Y_p' - 2Y_p &= -4C \sin 2x - 4D \cos 2x + A + 2C \cos 2x - 2D \sin 2x \\ &\quad - 2(Ax + B + C \sin 2x + D \cos 2x) \\ &= (-6C - 2D) \sin 2x + (-6D + 2C) \cos 2x \\ &\quad + A - 2Ax - 2B. \end{aligned}$$

$$= x + \sin 2x.$$

Equating coeff of like terms:

$$1: \quad A - 2B = 0$$

$$x: \quad -2A = 1$$

$$\sin 2x: \quad -6C - 2D = 1$$

$$\begin{aligned} \cos 2x: \quad -6D + 2C = 0 \quad \therefore C = 3D \quad \therefore -18D - 2D = 1 \\ \therefore D = -\frac{1}{20}, \quad C = \frac{3}{20}. \end{aligned}$$

$$\therefore A = -\frac{1}{2}, \quad B = \frac{1}{2}A = -\frac{1}{4}.$$

$$\therefore Y_p(x) = -\frac{1}{2}x - \frac{1}{4} - \frac{3}{20} \sin 2x - \frac{1}{20} \cos 2x.$$

$$P1154/11. \quad 4y'' + 5y' + y = e^x.$$

$$\text{Ch. eq. } 4m^2 + 5m + 1 = 0 \quad \therefore (4m+1)(m+1) = 0$$

$$\therefore m = -\frac{1}{4}, -1$$

$$\therefore Y_c = c_1 e^{-\frac{x}{4}} + c_2 e^{-x}$$

$$Y_p = Ae^x \quad \therefore Y_p' = Ae^x, \quad Y_p'' = Ae^x.$$

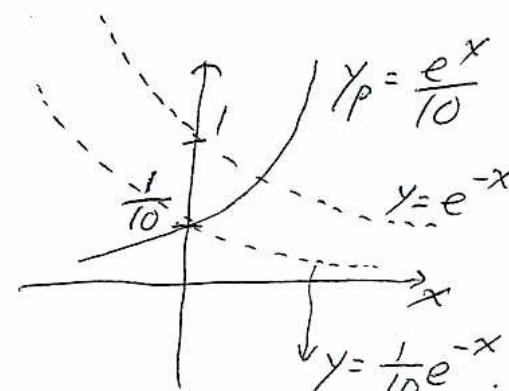
$$\therefore (4A + 5A + A)e^x = e^x.$$

$$\therefore 10Ae^x = e^x$$

$$\therefore A = \frac{1}{10}$$

$$\therefore Y_p = \frac{1}{10}e^x.$$

$$\text{Gen. Soln is } y = Y_c + Y_p.$$



D 1154 / 13.

$$y'' + 9y = e^{2x} + x^2 \sin x.$$

$$m^2 + 9 = 0 \quad \therefore m = \pm 3i$$

$$y = c_1 \cos 3x + c_2 \sin 3x,$$

where

$$\begin{aligned} c_1' &= \frac{\begin{vmatrix} 0 & \sin 3x \\ e^{2x} + x^2 \sin x & 3 \cos 3x \end{vmatrix}}{\begin{vmatrix} \cos 3x & \sin 3x \\ -3 \sin 3x & 3 \cos 3x \end{vmatrix}} = \frac{-\sin 3x (e^{2x} + x^2 \sin x)}{3 \cos^2 3x + 3 \sin^2 3x} \\ &= -\frac{\sin 3x (e^{2x} + x^2 \sin x)}{3}. \end{aligned}$$

$$\therefore c_1 = \frac{1}{3} \int -\sin 3x (e^{2x} + x^2 \sin x) dx + k_1.$$

$$c_2' = \frac{\begin{vmatrix} \cos 3x & 0 \\ -3 \sin 3x & e^{2x} + x^2 \sin x \end{vmatrix}}{3} = \frac{\cos 3x (e^{2x} + x^2 \sin x)}{3}$$

$$\therefore c_2 = \frac{1}{3} \int \cos 3x (e^{2x} + x^2 \sin x) dx + k_2.$$