

Instructions. You have 35 minutes to complete this quiz. Please provide detailed solutions for the exercises. Only complete answers with sufficient explanation are worth full credit. No open textbooks or notes are allowed.

1. [5 marks] Find a formula for $f^{-1}(x)$ and state the range and domain of $f^{-1}(x)$.

domain of $f(x) : [0, \infty)$
range of $f(x) : [-2, \infty)$
because $f'(x) \geq 0$ for $x \geq 0$.

$$f(x) = 3x^2 + 5x - 2, \quad x \geq 0$$

$$y = 3x^2 + 5x - 2$$

$$3x^2 + 5x - (2+y) = 0$$

$$x = \frac{-5 + \sqrt{25 + 12(2+y)}}{6}$$

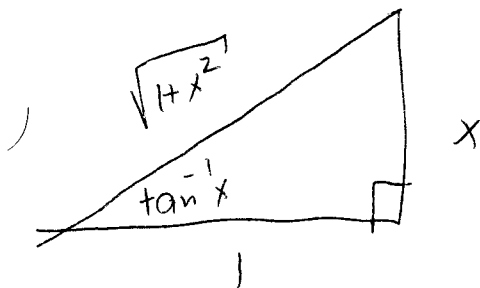
$$f^{-1}(x) = \frac{-5 + \sqrt{25 + 12(2+x)}}{6}$$

range of $f^{-1}(x) : [0, \infty)$
domain of $f^{-1}(x) : [-2, \infty)$

2. [5 marks] Given that $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ find the values of $\sin \theta$ and $\sec \theta$.

$$\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$$

$$\cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$



$$\begin{aligned} \sin \theta &= \sin\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right) = \frac{\frac{\sqrt{3}}{3}}{\sqrt{1 + \frac{1}{3}}} = \\ &= \frac{\frac{\sqrt{3}}{3}}{\frac{2}{\sqrt{3}}} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \cos\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right) = \frac{1}{\sqrt{1 + \frac{1}{3}}} = \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

3. [5 marks] Find $\frac{dy}{dx}$ by implicit differentiation.

$$\sin(x^2y^2) = x$$

$$\cos(x^2y^2)(2xy^2 + x^2 \cdot 2y \cdot y') = 1$$

$$y' \cdot 2x^2y \cos(x^2y^2) = 1 - 2xy^2 \cos(x^2y^2)$$

$$y' = \frac{1 - 2xy^2 \cos(x^2y^2)}{2x^2y \cos(x^2y^2)}$$

4. [5 marks] Solve the equation without a calculator.

$$\log_{10} x^{3/2} - \log_{10} \sqrt{x} = 5$$

$$\log_{10} \frac{x^{3/2}}{\sqrt{x}} = 5$$

$$\log_{10} x = 5$$

$$\underline{\underline{x = 10^5}}$$

- Marks:**
- 1)
 - 2)
 - 3)
 - 4)
 - Total: