

# SOLUTIONS TO QUIZ 3

1)

$$\int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}} dx$$

Discontinuity at  $\frac{\pi}{3}$ .

$$\lim_{a \rightarrow \frac{\pi}{3}^+} \int_a^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}} dx = \lim_{a \rightarrow \frac{\pi}{3}^+} \left[ \sqrt{1-2\cos x} \right]_a^{\pi/2}$$

$$\text{Let } u = 1-2\cos x \\ du = 2\sin x dx$$

$$\frac{1}{2} \int u^{-1/2} du \rightarrow u^{1/2} \rightarrow \sqrt{1-2\cos x}$$

$$\sqrt{1-2\cos\left(\frac{\pi}{2}\right)} - \lim_{a \rightarrow \frac{\pi}{3}^+} \sqrt{1-2\cos a}$$

$$\sqrt{1} - 0 = \boxed{1} \rightarrow \text{converges to } 1$$

$$2. \int_{-\infty}^{+\infty} \frac{x}{\sqrt{x^2+2}} dx = \int_{-\infty}^0 \frac{x}{\sqrt{x^2+2}} dx + \int_0^{\infty} \frac{x}{\sqrt{x^2+2}} dx$$

$$\lim_{a \rightarrow \infty} \int_{-a}^0 \frac{x}{\sqrt{x^2+2}} dx + \lim_{b \rightarrow \infty} \int_0^b \frac{x}{\sqrt{x^2+2}} dx$$

$$\Rightarrow \int \frac{x}{\sqrt{x^2+2}} dx \longrightarrow \frac{1}{2} \int u^{-1/2} du \rightarrow u^{1/2} = \sqrt{u} \\ \rightarrow \sqrt{x^2+2}$$

$$u = x^2 + 2 \\ du = 2x dx \\ \frac{1}{2} du = x dx$$

$$\text{So } \lim_{a \rightarrow -\infty} \left. \sqrt{x^2+2} \right|_{-a}^0 + \lim_{b \rightarrow \infty} \left. \sqrt{x^2+2} \right|_0^b$$

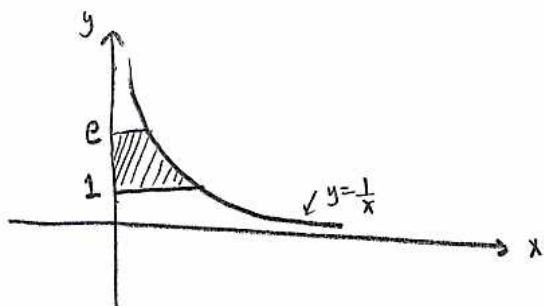
$$\left( \sqrt{2} - \lim_{a \rightarrow -\infty} \sqrt{a^2+2} \right) + \left( \lim_{b \rightarrow \infty} \sqrt{b^2+2} - \sqrt{2} \right)$$

$$-\infty + \infty$$

$\downarrow$                        $\downarrow$   
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Thus  $\int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2+2}} dx$  diverges.

3.  $x = \frac{1}{y} \rightarrow y = \frac{1}{x} \quad y=1 \quad y=e \quad x=0$



$$\int_1^e \frac{1}{y} dy = \ln|y| \Big|_1^e \\ = \ln e - \ln 1 \\ = \ln e - 0 \\ = 1$$