Formula Sheet for MATH253 (MIDTERM)

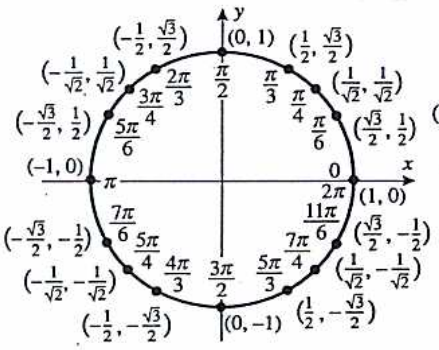
BASIC FUNCTIONS

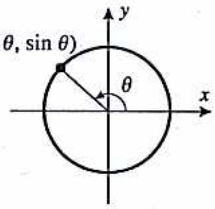
1. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$
2. $\int \frac{du}{u} = \ln|u| + C$
3. $\int e^u\,du = e^u + C$
4. $\int \sin u\,du = -\cos u + C$
5. $\int \cos u\,du = \sin u + C$
6. $\int \tan u\,du = \ln|\sec u| + C$
7. $\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1-u^2} + C$
8. $\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$
9. $\int \tan^{-1} u\,du = u\tan^{-1} u - \ln\sqrt{1+u^2} + C$
10. $\int a^u\,du = \frac{a^u}{\ln a} + C$
11. $\int \ln u\,du = u\ln u - u + C$
12. $\int \cot u\,du = \ln|\sin u| + C$
13. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
    $= \ln\left|\tan\left(\frac{1}{4}\pi + \frac{1}{2}u\right)\right| + C$
14. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
    $= \ln\left|\tan\frac{1}{2}u\right| + C$
15. $\int \cot^{-1} u\,du = u\cot^{-1} u + \ln\sqrt{1+u^2} + C$
16. $\int \sec^{-1} u\,du = u\sec^{-1} u - \ln|u + \sqrt{u^2-1}| + C$
17. $\int \csc^{-1} u\,du = u\csc^{-1} u + \ln|u + \sqrt{u^2-1}| + C$

POWERS OF TRIGONOMETRIC FUNCTIONS

26. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$
27. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$
28. $\int \tan^2 u\,du = \tan u - u + C$
29. $\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$
30. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u\sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du$
31. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$
32. $\int \cot^2 u\,du = -\cot u - u + C$
33. $\int \sec^2 u\,du = \tan u + C$
34. $\int \csc^2 u\,du = -\cot u + C$
35. $\int \cot^n u\,du = -\frac{1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du$
36. $\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u\tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$
37. $\int \csc^n u\,du = -\frac{1}{n-1}\csc^{n-2} u\cot u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du$

TRIGONOMETRY REVIEW





PYTHAGOREAN IDENTITIES

$\sin^2\theta + \cos^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

SIGN IDENTITIES

$\sin(-\theta) = -\sin\theta$ $\quad \cos(-\theta) = \cos\theta$ $\quad \tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$ $\quad \sec(-\theta) = \sec\theta$ $\quad \cot(-\theta) = -\cot\theta$

COMPLEMENT IDENTITIES

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

SUPPLEMENT IDENTITIES

$\sin(\pi - \theta) = \sin\theta$ $\quad \cos(\pi - \theta) = -\cos\theta$ $\quad \tan(\pi - \theta) = -\tan\theta$
$\csc(\pi - \theta) = \csc\theta$ $\quad \sec(\pi - \theta) = -\sec\theta$ $\quad \cot(\pi - \theta) = -\cot\theta$
$\sin(\pi + \theta) = -\sin\theta$ $\quad \cos(\pi + \theta) = -\cos\theta$ $\quad \tan(\pi + \theta) = \tan\theta$
$\csc(\pi + \theta) = -\csc\theta$ $\quad \sec(\pi + \theta) = -\sec\theta$ $\quad \cot(\pi + \theta) = \cot\theta$

ADDITION FORMULAS

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

DOUBLE-ANGLE FORMULAS

$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$

HALF-ANGLE FORMULAS

$\sin^2\frac{\alpha}{2} = \frac{1 - \cos\alpha}{2}$ $\quad \cos^2\frac{\alpha}{2} = \frac{1 + \cos\alpha}{2}$