

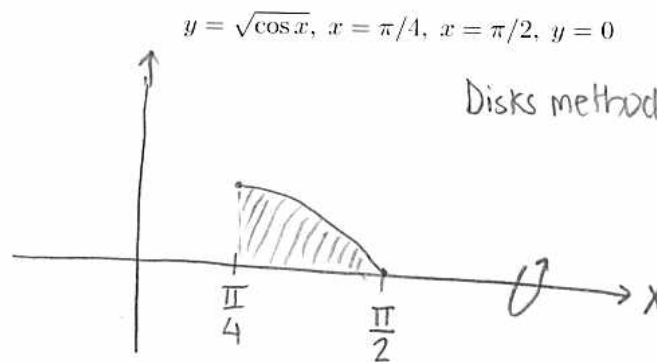
SOLUTIONS

Quiz 4 for Math253
August 8, 2006

Name:

Instructions. You have 35 minutes to complete this quiz. Please provide detailed solutions for the exercises. Only complete answers with sufficient explanation are worth full credit. No open textbooks or notes are allowed.

1. [7 marks] Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x -axis.

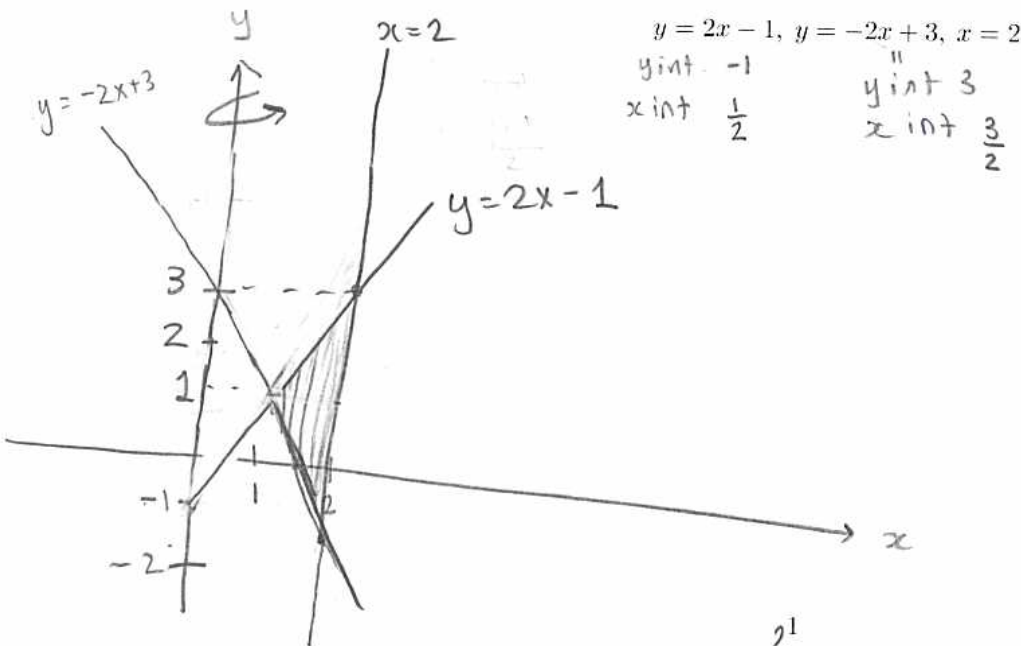


$$V = \pi \int_{\pi/4}^{\pi/2} [\sqrt{\cos x} - 0]^2 dx$$

$$= \pi \int_{\pi/4}^{\pi/2} \cos x dx$$

$$= \pi \sin x \Big|_{\pi/4}^{\pi/2}$$

2. [7 marks] Find the volume of the solid that results when the region enclosed by the given curves is revolved about the y -axis.



$$= \pi \left[1 - \frac{1}{\sqrt{2}} \right]$$

$$= \pi \left[1 - \frac{\sqrt{2}}{2} \right] \text{ OR}$$

cylindrical shells.

$$V = 2\pi \int_{1/2}^2 x (2x - 1 - (-2x + 3)) dx$$

$$\frac{20\pi}{3}$$

$$\frac{2\pi(10)}{3}$$

$$V = 2\pi \int_{1/2}^2 x(4x - 4) dx \rightarrow 2\pi \int_{1/2}^2 (4x^2 - 4x) dx \rightarrow 2\pi \left(\frac{4}{3}x^3 - 2x^2 \right) \Big|_{1/2}^2$$

UofC ID:..... *Solution Key*

3. [6 marks] Find the arclength of the curve over the stated interval.

$$x = (1+t)^2, y = (1+t)^3, (0 \leq t \leq 1)$$

$$\frac{dx}{dt} = 2(t+1), \frac{dy}{dt} = 3(1+t)^2$$

$$L = \int_0^1 \sqrt{4(1+t)^2 + 9(1+t)^4} dt = \int_0^1 (1+t) \sqrt{4 + 9(1+t)^2} dt = \textcircled{*}$$

$$u = 4 + 9(1+t)^2$$

$$du = 18(1+t) dt$$

$$u(0) = 13$$

$$u(1) = 40$$

Marks:

1:.....

2:.....

3:.....

TOTAL:.....

$$\textcircled{*} = \frac{1}{18} \int_{13}^{40} \sqrt{u} du = \frac{1}{27} \left[\sqrt{u}^3 \right]_{13}^{40} = \frac{1}{27} \left(40^{3/2} - 13^{3/2} \right)$$