SOLUTIONS

Quiz 5 for Math253
August 15, 2006

Name:......................

Instructions. You have 35 minutes to complete this quiz. Please provide detailed solutions for the exercises. Only complete answers with sufficient explanation are worth full credit. No open textbooks or notes are allowed.

1. [6 marks] Find the Taylor polynomial of order four about $x = x_0$ for the following function.

$$\sin(\pi x),\ x_0 = \frac{1}{2}$$

$$f(x) = \sin(\pi x) \qquad f(\tfrac{1}{2}) = \sin\tfrac{\pi}{2} = 1$$

$$f'(x) = \pi\cos(\pi x) \qquad f'(1/2) = \pi\cos\tfrac{\pi}{2} = 0$$

$$f''(x) = -\pi^2\sin(\pi x) \qquad f''(1/2) = -\pi^2$$

$$f'''(x) = -\pi^3\cos(\pi x) \qquad f'''(1/2) = 0$$

$$f^{(4)}(x) = \pi^4\sin(\pi x) \qquad f^{(4)}(1/2) = \pi^4$$

$$P_4(x) = 1 + \underbrace{\cancel{\frac{0(x-\frac{1}{2})^1}{1!}}}_{0} - \frac{\pi^2(x-\frac{1}{2})^2}{2!} + \underbrace{\cancel{\frac{0(x-\frac{1}{2})^3}{3!}}}_{0} + \frac{\pi^4(x-\frac{1}{2})^4}{4!}$$

$$= 1 - \frac{\pi^2(x-\frac{1}{2})^2}{2} + \frac{\pi^4(x-\frac{1}{2})^4}{24} \quad 2$$

2. [6 marks] Find the first four nonzero terms of the Maclaurin series of the following function.

$$e^{-x^2}\cos x$$

(1)

$x_0 = 0$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \qquad (1)\quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$e^{-x^2} = \sum_{k=0}^{\infty} \frac{(-x^2)^k}{k!} \quad (1)$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{k!}$$

$$e^{-x^2}\cos x = \left(\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{k!}\right)\left(\sum_{k=0}^{\infty} \frac{(-1)^{2k} x^{2k}}{(2k)!}\right)$$

(1)

$$= \left(1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots\right)\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right)$$

(1)

$$= 1 + \left(\frac{-1}{2!} - \frac{1}{1!}\right)x^2 + \left(\frac{1}{4!} + \frac{1}{2!\,1!} + \frac{1}{2!}\right)x^4 + \left(\frac{-1}{6!} - \frac{1}{1!\,4!} - \frac{1}{2!\,2!} - \frac{1}{3!}\right)x^6 + \cdots$$

(1)

$$\boxed{= 1 - \frac{3}{2}x^2 + \frac{25}{24}x^4 - \frac{331}{720}x^6 + \cdots}$$

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3. [8 marks] Solve the differential equation.

$$\frac{dy}{dx} + 4y = e^{-3x}$$

$y' + 4y = e^{-3x}$ → Linear. $y' + p(x)y = q(x)$

$p(x) = 4$

$q(x) = e^{-3x}$

$\mu(x) = e^{\int p(x)dx} = e^{\int 4dx} = e^{4x}$ 3

Solution

$y\mu = \int q\mu\, dx$ 1

$ye^{4x} = \int e^{4x} e^{-3x} dx \rightarrow ye^{4x} = \int e^{x} dx$ 2

$\rightarrow ye^{4x} = e^{x} + C$

$\boxed{y = e^{-3x} + Ce^{-4x}}$ 2

Marks:

1:.............

2:.............

3:.............

TOTAL:.............