Solution

Name: ______________________ ID Number: ____________

UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 253 — L03 WINTER 2004

QUIZ #3b [04-03-09(Tues)]

Attempt all problems. Each problem: 10 marks. Total: 30 marks (best 3 out of 4 problems).

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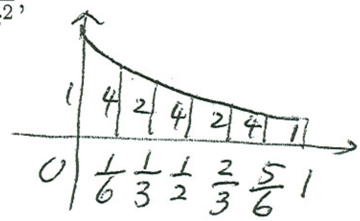
1. Using Simpson's rule and $n = 6$ subdivisions, evaluate $\int_0^1 \frac{dx}{6+x^2}$, up to 3 decimal places. $\Delta x = \frac{1}{6}$.



$$\frac{\Delta x}{3}\left[f(0)+4\left\{f(\tfrac{1}{6})+f(\tfrac{1}{2})+f(\tfrac{5}{6})\right\} +2\left\{f(\tfrac{1}{3})+f(\tfrac{2}{3})\right\} + f(1)\right]$$

$$= \frac{1}{18}\left[\frac{1}{6}+4\left\{\frac{1}{6+\frac{1}{36}}+\frac{1}{6+\frac{1}{4}}+\frac{1}{6+\frac{25}{36}}\right\} +2\left\{\frac{1}{6+\frac{1}{9}}+\frac{1}{6+\frac{4}{9}}\right\}+\frac{1}{7}\right]$$

$$\simeq 0.158$$

2. Evaluate the improper integral $\int_1^2 \frac{dx}{(x-1)^{1/3}}$ (improper at $x=1$)

$$= \lim_{\delta\to 0^+} \int_{1+\delta}^{2} \frac{dx}{(x-1)^{1/3}}$$

$$= \text{''} \quad \frac{3}{2}(x-1)^{2/3}\Big]_{1+\delta}^{2}$$

$$= \text{''} \quad \frac{3}{2}(1)^{2/3} - \frac{3}{2}(\delta)^{2/3} = \frac{3}{2}.$$

3. Find the arc length of the curve: $x = \frac{1}{3}(y^{3/2} - 3y^{1/2}), \quad 0 \le y \le 9.$

$$\frac{dx}{dy} = \frac{1}{2}y^{1/2} - \frac{1}{2y^{1/2}} = \frac{1}{2}\left(y^{1/2} - \frac{1}{y^{1/2}}\right)$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}\left(y - 2 + \frac{1}{y}\right)$$

$$1+\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}\left(y + 2 + \frac{1}{y}\right)$$

$$= \frac{1}{4}\left(y^{1/2} + \frac{1}{y^{1/2}}\right)^2$$

$$\therefore \sqrt{1+x'^2} = \frac{1}{2}\left(y^{1/2} + \frac{1}{y^{1/2}}\right)$$

$$s = \frac{1}{2}\int_0^9 y^{1/2} + y^{-\frac{1}{2}}\, dy \qquad \text{(improper at } y=0\text{)}$$

$$= \frac{1}{2}\left(\frac{2}{3}y^{3/2} + 2y^{\frac{1}{2}}\right)\Big|_0^9$$

$$= \frac{1}{2}\left(\frac{2}{3}\cdot 27 + 2\cdot 3\right) = 9+3 = 12.$$

4. Find the area of the surface obtained by rotating about the x-axis the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}, \quad 1 \le x \le 2.$

$$y' = x^3 - \frac{1}{4x^3}$$

$$y'^2 = \left(x^3 - \frac{1}{4x^3}\right)^2$$

$$= x^6 - \frac{1}{2} + \frac{1}{16x^6}$$

$$1+y'^2 = x^6 + \frac{1}{2} + \frac{1}{16x^6}$$

$$= \left(x^3 + \frac{1}{4x^3}\right)^2$$

$$S = 2\pi\int_1^2 y\sqrt{1+y'^2}\, dx$$

$$= 2\pi\int_1^2 \left(\frac{x^4}{4} + \frac{1}{8x^2}\right)\left(x^3 + \frac{1}{4x^3}\right) dx$$

$$= 2\pi\int_1^2 \frac{x^4}{4}\left(x^3 + \frac{1}{4x^3}\right) + \frac{1}{8x^2}\left(x^3 + \frac{1}{4x^3}\right) dx$$

$$= 2\pi\int_1^2 \frac{x^7}{4} + \frac{1}{16} + \frac{x}{8} + \frac{1}{32x^5}\, dx$$

$$= 2\pi\left[\frac{x^8}{32} + \frac{x}{16} + \frac{x^2}{16} - \frac{1}{128x^4}\right]_1^2$$

$$= 2\pi\left(2^3 + \cancel{\frac{1}{8}} + \frac{1}{4} - \frac{1}{128\cdot 2^4} - \left(\frac{1}{32} + \cancel{\frac{1}{16}} + \cancel{\frac{1}{16}} - \frac{1}{128}\right)\right)$$

$$= \pi\left(16 + \frac{1}{2} - \frac{1}{128\cdot 8} - \frac{1}{16} + \frac{1}{64}\right).$$