Operator Method of finding x_p.

1. $\frac{1}{D}\cdot 1 = \int 1\,dt = t$

   $\frac{1}{D^2}\cdot 1 = \int t\,dt = \frac{t^2}{2} \quad \ldots\ldots$

   $\frac{1}{D}t = \int t\,dt = \frac{t^2}{2}$.

   $\frac{1}{D^2}t = \int \frac{t^2}{2}dt = \frac{t^3}{2\cdot 3} = \frac{t^3}{3!} \quad \ldots$

2. (i) $\frac{1}{1+D}t^n = (1+D)^{-1}t^n = (1-D+D^2-D^3+\cdots)t^n$

   (ii) $\frac{1}{1-D}t^n = (1-D)^{-1}t^n = (1+D+D^2+D^3+\cdots)t^n$

3. (i) $\frac{1}{b+D}e^{at} = \frac{e^{at}}{b+a}$ if $b+a\neq 0$ (replace D by a)

   (ii) $\frac{1}{b+D}e^{at} = e^{at}\frac{1}{b+(D+a)}\cdot 1$ if $b+a=0$ $(D\to D+a)$

4. $\frac{1}{1+D}te^{at} = e^{at}\frac{1}{1+(D+a)}\cdot t$ $(D\to D+a)$

5. $\frac{1}{1+D}\cos at = \frac{1}{1+D}\,\frac{1-D}{1-D}\cos at$

   $= \frac{1-D}{1-D^2}\cos at$

   $= \frac{1-D}{1-(-a^2)}\cos at \quad [D^2\to -a^2]$

$$= \frac{1-D}{1+a^2} \cos at$$

$$= \frac{\cos at + a \sin at}{1+a^2}$$

6. $e^{iat} = \cos at + i \sin at$

$(\therefore \cos at = Re\, e^{iat}$

$\quad \& \sin at = Im\, e^{iat})$

$$\frac{1}{D^2+a^2} \cos at = Re \frac{1}{D^2+a^2} e^{iat}$$

$$= Re\, e^{iat} \frac{1}{(D+ia)^2+a^2} \cdot 1 \quad \text{, using 3(ii)}$$

$$= Re\, e^{iat} \frac{1}{D^2+2iaD} \cdot 1$$

$$= Re\, e^{iat} \frac{1}{2iaD(1+\frac{D}{2ia})} 1$$

$\because i^2 = -1$

$\therefore i = \frac{-1}{i}$

$\therefore -i = \frac{1}{i}$

$$= Re\, e^{iat} \frac{-i}{2aD(1+\frac{D}{2ia})} \cdot 1$$

$$= Re\, e^{iat} \frac{-i}{2aD} \left(1 - \frac{D}{2ia} + \ldots\right) 1 \text{, using 2(i).}$$

$$= \mathrm{Re}\, e^{iat} \frac{-i}{2aD} \cdot 1$$

$$= \mathrm{Re}\, e^{iat} \frac{-i}{2a} t$$

$$= \mathrm{Re} \frac{-it}{2a} (\cos at + i \sin at)$$

$$= \mathrm{Re} \frac{-it}{2a} \cos at + \frac{t}{2a} \sin at$$

$$= \frac{t}{2a} \sin at$$