

A SHORT TABLE OF INDEFINITE INTEGRALS

I. Basic Functions

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| 1. $\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$ | 5. $\int \sin x dx = -\cos x + C$ |
| 2. $\int \frac{1}{x} dx = \ln x + C$ | 6. $\int \cos x dx = \sin x + C$ |
| 3. $\int a^x dx = \frac{1}{\ln a}a^x + C$ | 7.(a) $\int \tan x dx = -\ln \cos x + C$ |
| 4. $\int \ln x dx = x \ln x - x + C, \quad x > 0$ | 7.(b) $\int \cot x dx = \ln \sin x + C$ |
| | 7.(c) $\int \sec^2 x dx = \tan x + C$ |
| | 7.(d) $\int \csc^2 x dx = -\cot x + C$ |

II Products of e^x , $\cos x$, and $\sin x$

8. $\int e^{ax} \sin(bx) dx = \frac{1}{a^2+b^2}e^{ax}[a \sin(bx) - b \cos(bx)] + C$
9. $\int e^{ax} \cos(bx) dx = \frac{1}{a^2+b^2}e^{ax}[a \cos(bx) + b \sin(bx)] + C$
10. $\int \sin(ax) \sin(bx) dx = \frac{1}{b^2-a^2}[a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, \quad a \neq b$
11. $\int \cos(ax) \cos(bx) dx = \frac{1}{b^2-a^2}[b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)] + C, \quad a \neq b$
12. $\int \sin(ax) \cos(bx) dx = \frac{1}{b^2-a^2}[b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x , $\cos x$, $\sin x$

13. $\int x^n \ln x dx = \frac{1}{n+1}x^{n+1} \ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1, \quad x > 0$
14. $\begin{aligned} \int p(x)e^{ax} dx &= \frac{1}{a}p(x)e^{ax} - \frac{1}{a} \int p'(x)e^{ax} dx \\ &= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \dots \\ &\quad (+ - + - \dots) \quad (\text{signs alternate}) \end{aligned}$
15. $\begin{aligned} \int p(x) \sin ax dx &= -\frac{1}{a}p(x) \cos ax + \frac{1}{a} \int p'(x) \cos ax dx \\ &= -\frac{1}{a}p(x) \cos ax + \frac{1}{a^2}p'(x) \sin ax + \frac{1}{a^3}p''(x) \cos ax - \dots \\ &\quad (- + + - + + \dots) \quad (\text{signs alternate in pairs after first term}) \end{aligned}$
16. $\begin{aligned} \int p(x) \cos ax dx &= \frac{1}{a}p(x) \sin ax - \frac{1}{a} \int p'(x) \sin ax dx \\ &= \frac{1}{a}p(x) \sin ax + \frac{1}{a^2}p'(x) \cos ax - \frac{1}{a^3}p''(x) \sin ax - \dots \\ &\quad (+ + - - + + - \dots) \quad (\text{signs alternate in pairs}) \end{aligned}$

IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x dx = -\frac{1}{n}(\sin^{n-1} x) \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx, \quad n \text{ positive}$
18. $\int \cos^n x dx = \frac{1}{n}(\cos^{n-1} x) \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx, \quad n \text{ positive}$
19. $\int \frac{1}{\sin^m x} dx = \frac{-1}{(m-1)} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} dx, \quad m \neq 1, \quad m \text{ positive}$
20. $\int \frac{1}{\sin x} dx = \int \csc x dx = \ln[\csc x - \cot x] + C$
21. $\int \frac{1}{\cos^m x} dx = \frac{1}{(m-1)} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} dx, \quad m \neq 1, \quad m \text{ positive}$
22. $\int \frac{1}{\cos x} dx = \int \sec x dx = \ln[\sec x + \tan x] + C$
23. $\int \sin^m x \cos^n x dx : \text{ If } m \text{ is odd, let } w = \cos x. \text{ If } n \text{ is odd, let } w = \sin x.$

If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C, \quad a \neq 0$
25. $\int \frac{bx+c}{x^2+a^2} dx = \frac{b}{2} \ln|x^2+a^2| + \frac{c}{a} \arctan \frac{x}{a} + C, \quad a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)} dx = \frac{1}{a-b} (\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$
27. $\int \frac{cx+d}{(x-a)(x-b)} dx = \frac{1}{a-b} [(ac+d) \ln|x-a| - (bc+d) \ln|x-b|] + C, \quad a \neq b$

VI: Integrands involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $a > 0$

28. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a} + C$
29. $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}| + C$
30. $\int \sqrt{a^2 \pm x^2} dx = \frac{1}{2} \left(x \sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} dx \right) + C$
31. $\int \sqrt{x^2 - a^2} dx = \frac{1}{2} \left(x \sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} dx \right) + C$