

04-02-9 (Mon.)

Solution  
MATH 253 (L03)  
QUIZ 2

Name: \_\_\_\_\_

ID: \_\_\_\_\_

Attempt all prob.

Max: 30 marks (best 3 out of 4 problems)

1. Evaluate

(i)  $\frac{d}{dx} \int_{2x}^{x^2} \sqrt{2-x^2} dx$

$$= \sqrt{2-x^2} \cdot 2x - \sqrt{2-4x^2} \cdot 2.$$

(ii)  $\int \frac{e^x + e^{-x}}{(e^x - e^{-x})^2} dx$

$$= -(e^x - e^{-x})^{-1} + C$$

$$= \frac{-1}{e^x - e^{-x}} + C$$

Or let  $u = e^x - e^{-x}$ .

2. Evaluate (i)  $\int x(1-2x)^{1/2} dx$ .

Let  $u = 1-2x$  [ $x = \frac{1}{2}(1-u)$ ]

$du = -2dx$

$\therefore dx = -\frac{1}{2} du$

$$\int \frac{1}{2}(1-u)u^{1/2} \cdot \frac{-1}{2} du$$

$$= -\frac{1}{4} \int u^{1/2} - u^{3/2} du$$

$$= -\frac{1}{4} \left[ \frac{2}{3} u^{3/2} - \frac{2}{5} u^{5/2} \right] + C$$

$$= -\frac{1}{4} \left( \frac{2}{3} (1-2x)^{3/2} - \frac{2}{5} (1-2x)^{5/2} \right) + C$$

or  $\frac{1}{10}(1-2x)^{5/2} - \frac{1}{6}(1-2x)^{3/2} + C$

(ii)  $\int \cos^3 2x dx$ .

$$= \int \cos 2x (1 - \sin^2 2x) dx$$

$$= \int \cos 2x - \cos 2x \sin^2 2x dx$$

$$= \frac{1}{2} \sin 2x - \frac{1}{3 \cdot 2} \sin^3 2x + C$$

Or, by parts.

3. Evaluate

$$(i) \int (1-x)^2 \ln x \, dx \quad (\text{by parts})$$

$$= \frac{-1}{3} (1-x)^3 \ln x + \frac{1}{3} \int (1-x)^3 \cdot \frac{1}{x} \, dx$$

$$= \text{"} + \frac{1}{3} \int \frac{1-3x+3x^2-x^3}{x} \, dx$$

$$= \text{"} + \frac{1}{3} \int \left( \frac{1}{x} - 3 + 3x - x^2 \right) \, dx$$

$$= \text{"} + \frac{1}{3} \left[ \ln|x| - 3x + \frac{3x^2}{2} - \frac{x^3}{3} \right] + C$$

$$(ii) \int \frac{x}{(1-x)(1+x)} \, dx$$

$$\frac{x}{(1-x)(1+x)} = \frac{A}{1-x} + \frac{B}{1+x}$$

$$x = A(1+x) + B(1-x)$$

$$\text{Let } x=1 \quad 1 = A \cdot 2 \quad \therefore A = \frac{1}{2}$$

$$x=-1 \quad -1 = B \cdot 2 \quad \therefore B = -\frac{1}{2}$$

$$\int \frac{1/2}{1-x} - \frac{1/2}{1+x} \, dx$$

$$= -\frac{1}{2} \ln|1-x| - \frac{1}{2} \ln|1+x| + C$$

$$\text{or } -\frac{1}{2} \ln \left| \frac{1-x}{1+x} \right| + C$$

$$\text{or } \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C$$

4. Evaluate  $I = \int \frac{1-x^2}{x(x-5)(x+1)} \, dx.$

$$\frac{1-x^2}{x(x-5)(x+1)} = \frac{A}{x} + \frac{B}{x-5} + \frac{C}{x+1}$$

$$1-x^2 = A(x-5)(x+1) + Bx(x+1) + Cx(x-5)$$

Let

$$x=5 \quad 1-25 = -24 = B \cdot 5 \cdot 6 \quad \therefore B = -\frac{4}{5}$$

$$x=-1 \quad 0 = C(-1)(-6) \quad \therefore C = 0$$

$$x=0 \quad 1 = A(-5) \quad \therefore A = -\frac{1}{5}$$

$$\therefore I = \int \frac{-\frac{1}{5}}{x} - \frac{\frac{4}{5}}{x-5} \, dx$$

$$= -\frac{1}{5} \ln|x| - \frac{4}{5} \ln|x-5| + C.$$