SOLUTION

04-02-10 (Tue.) MATH 253 (L03) Name:

QUIZ 2. ID: _____

Attempt all prob.

Max: 30 marks (best 3 out of 4 prob.)

1. Evaluate

(i) $\frac{d}{dx}\int_{-x}^{3x}\sqrt{x^2-4}\,dx$

$= \sqrt{9x^2-4}\cdot 3 - (-1)\sqrt{x^2-4}$

$= 3\sqrt{9x^2-4} + \sqrt{x^2-4}$

(ii) $\int \frac{e^x-e^{-x}}{(e^x+e^{-x})^3}\,dx$

$= \frac{1}{-2}(e^x+e^{-x})^{-2} + c$

$= \frac{1}{-2(e^x+e^{-x})^2} + c$

Or

let $u = e^x+e^{-x}$.

2. Evaluate (i) $\int x(3x-1)^{1/3}\,dx$

Let $t = 3x-1$ $\left(x = \frac{1}{3}(t+1)\right)$

$dt = 3\,dx$

$\frac{dt}{3} = dx$.

$\int \frac{1}{3}(t+1)\,t^{1/3}\,\frac{dt}{3}$

$= \frac{1}{9}\int t^{4/3} + t^{1/3}\,dt$

$= \frac{1}{9}\left[\frac{3}{7}t^{7/3} + \frac{3}{4}t^{4/3}\right] + C$

$= \frac{1}{21}(3x-1)^{7/3} + \frac{1}{12}(3x-1)^{4/3} + C$

(ii) $\int \sin^3 3x\,dx$

$\int \sin 3x\,(1-\cos^2 3x)\,dx$

$= \int \sin 3x - \sin 3x\cos^2 3x\,dx$

$= -\frac{1}{3}\cos 3x + \frac{1}{3\cdot 3}\cos^3 3x + C$.

Or, by parts.

3. Evaluate (i) $\int (1+2x)^2 \ln x \, dx$ (ii) $\int \frac{x-1}{x(4x+1)} dx$

$= \frac{1}{6}(1+2x)^3 \ln x - \int \frac{1}{6}(1+2x)^3 \frac{1}{x} dx$

$= \quad " \quad - \frac{1}{6}\int \frac{1+6x+12x^2+8x^3}{x} dx$

$= \quad " \quad - \frac{1}{6}\int \frac{1}{x} + 6 + 2x + 8x^2 \, dx$

$= \quad " \quad - \frac{1}{6}\left[\ln|x| + 6x + x^2 + \frac{8x^3}{3}\right] + C$

Note: $(1+2x)^3 = 1+6x+12x^2+8x^3$

$\frac{x-1}{x(\,)} = \frac{A}{x} + \frac{B}{4x+1}$

$x-1 = A(4x+1) + Bx$

Set

$x=0 \quad -1 = A \quad \therefore A = -1$

$x=-\frac{1}{4} \quad -\frac{5}{4} = B\left(-\frac{1}{4}\right)$

$\therefore B = 5$

$\therefore \int \frac{-1}{x} + \frac{5}{4x+1} dx$

$= -\ln|x| + \frac{5}{4}\ln|4x+1| + C$

4. Evaluate $I = \int \frac{x^2-1}{x(1+x)(5-x)} dx$

$\frac{x^2-1}{x(\,)(\,)} = \frac{A}{x} + \frac{B}{1+x} + \frac{C}{5-x}$

$x^2-1 = A(1+x)(5-x) + Bx(5-x) + Cx(1+x)$

Set

$x=0 \quad -1 = A \cdot 5 \quad \therefore A = -1/5$

$x=-1 \quad 0 = B(-1)(6) \quad \therefore B = 0$

$x=5 \quad 24 = C \cdot 5 \cdot 6 \quad \therefore C = 4/5$

$\therefore I = \int \frac{-\frac{1}{5}}{x} + \frac{\frac{4}{5}}{5-x} dx$

$= -\frac{1}{5}\ln|x| - \frac{4}{5}\ln|5-x| + C_1$