

UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATHEMATICS 253 — L03 WINTER 2004
MIDTERM REVIEW

Midterm exam: March 5, 2004

1. Find the inverse functions of the following functions and their domain and range

(a) $f(x) = \frac{1-x}{3x-2}$.

(b) $f(x) = \sin(3x+2)$.

2. Evaluate:

(a) $\int \sin^{-1} 2x \, dx$.

(b) $\int_0^{\pi/4} x \sin 2x \, dx$.

(c) $\int_1^2 x(\ln x)^2 \, dx$.

(d) $\int \frac{x}{2x^2 - x + 2} \, dx$.

(e) $\int \frac{x+1}{2x^2 - x - 1} \, dx$.

(f) $\int \frac{x^2}{\sqrt{4x^2 + 9}} \, dx$.

(g) $\int_0^1 x^2 e^{-2x} \, dx$.

3. Evaluate the improper integrals

(a) $\int_0^2 \frac{dx}{x^{1/2}}$.

(b) $\int_1^{\infty} \ln x \, dx$.

(c) $\int_1^{\infty} \frac{\ln x}{x^2} \, dx$.

(d) $\int_0^1 x^2 \ln x \, dx$.

4. Find $F'(x)$ and $F''(x)$, if

(a) $F(x) = \int_{-x}^{2x} \sin 6x \, dx$

(b) $F(x) = \int_{2x}^{x^2} \ln(x+1) \, dx.$

5. Find the arclength of the curve over the given interval

(a) $y = \frac{1}{2}(e^x + e^{-x})$ from $x = 0$ to $x = 1$.

(b) $x = \frac{y^4}{8} + \frac{1}{4y^2}$ from $y = 1$ to $y = 4$.

(c) $y = \sqrt{9 - 4x^2}$ from $x = 0$ to $x = 1$.

(d) $y = \cos 2t$, $x = \sin 2t$ from $t = 0$ to $t = \frac{\pi}{4}$.

HINT: $\int_0^{\pi/4} \sqrt{x'^2 + y'^2} \, dt.$