

**FINAL Handout**  
**MATH 253**

1. Is the integral convergent? If so evaluate it.

(a) (a)  $\int_{-\infty}^0 x e^x dx$       (b)  $\int_0^e \ln x dx$

2. Given  $f(x) = \frac{x^3}{x+1}$  on the interval  $(-\infty, -3]$  show that the inverse exists and find its domain and range

3. Derive the formula for the volume of

(a) a sphere with radius  $R$        $(V = \frac{4}{3}\pi R^3)$

(b) a cone with radius  $R$  and the height  $H$        $(V = \frac{1}{3}\pi R^2 H)$

4. Derive the formula for circumference of a circle with radius  $R$ . ( $c = 2\pi R$ )

5. Find the domain of definition of  $f(x) = \sqrt{9-x^2}$  and then find the antiderivative  $F(x) = \int f(x) dx$  - NOT using Tables. (Area of a circle)

6. Approximate  $\arcsin \frac{1}{3}$  using the Taylor polynomial of third degree  $T_3$  centered at 0.

7. Find the general solution of  $x^2 y' - 4y = x^3 \cdot \ln x \cdot e^{-4/x}$ .

8. Solve the initial value problem

$$y'' + 6y' + 9y = 18x^2, \quad y(0) = \frac{1}{3}, \quad y'(0) = 3.$$

9. Find the general solution of the differential equation

$$y'' + 9y = 10 \sin 2x + e^{-x}$$

10. Find the domain and antiderivative of the following functions:

(a)  $x \arcsin(2x)$

(b)  $\frac{x^2 + 2}{x - x^2}$

(c)  $x \ln(2x + 3)$

(d)  $\frac{1}{\sqrt{e^x + 1}}$