MATH 253 HANDOUT #4 SOLUTION

A For 1)

For $\int_0^2 \frac{1}{1+x^2}\,dx$ and $n = 4$

The length of the subintervals is $\triangle x = h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$, so the points of partition are:
$a = x_0 = 0, x_1 = \frac{1}{2}, x_2 = 1, x_3 = \frac{3}{2}, x_4 = b = 2$
then the values of $y = f(x) = \frac{1}{x^2+1}$ are
y$_0 = 1, y_1 = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}, y_2 = \frac{1}{2}, y_3 = \frac{1}{1+\frac{9}{4}} = \frac{4}{13}, y_4 = \frac{1}{5},$
so $T_4 = \frac{1}{2}\left[\frac{1}{2} + \frac{4}{5} + \frac{1}{2} + \frac{4}{13} + \frac{1}{10}\right] = \frac{1}{2}\left[1 + \frac{9}{10} + \frac{4}{13}\right] = \frac{1}{2}\cdot\frac{130+117+40}{130} =$
$= \frac{287}{260} = 1.1038462$ By direct integration:

$$\int_0^2 \frac{1}{1+x^2}\,dx = [\arctan x]_0^2 = \arctan 2 - \arctan 0 = \arctan 2 = 1.1071487,$$

so we got an approximation of $\arctan 2 \doteq 1.1038462$

For 2)

First,sketch the region :hyperbola $y = \frac{6}{x}$.... top
$y = 2$... bottom $2 \le x \le 3$
y-axis→" shells"→ dx

$$V = 2\pi \int_2^3 RH\,dx, \text{ where } R = x, \text{and } H = \text{"}hyperbola - line\text{"} = \tfrac{6}{x} - 2,$$

$$V = 2\pi \int_2^3 x \cdot \left(\tfrac{6}{x} - 2\right) dx = 2\pi \cdot 6\,(3-2) - 2\pi \int_2^3 2x dx =$$

$= 12\pi - 2\pi\,[x^2]_2^3 = 12\pi - 2\pi\,(9-4) = 2\pi.$

For 3)

For $y = \ln(\sin x)$ between $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$

$y' = \dfrac{\cos x}{\sin x}$ $\quad 1 + (y')^2 = \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x} = \csc^2 x$

then $s = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1+(y')^2}dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc x dx = [\ln|\csc x - \cot x|]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \ln 1 - \ln\left(\sqrt{2}-1\right) =$

$= \ln\frac{1}{\sqrt{2}-1} = \ln\left(\sqrt{2}+1\right).$

B For 1)

For $\int_1^3 \frac{1}{x-4}\,dx$ and $n = 3$

the length of the subintervals is $\triangle x = h = \frac{b-a}{n} = \frac{3-1}{3} = \frac{2}{3}$
partition points are: $a = x_0 = 1, x_1 = \frac{5}{3}, x_2 = \frac{7}{3}, b = x_3 = \frac{9}{3} = 3$, and midpoints are:
$m_1 = \frac{4}{3}, m_2 = \frac{6}{3} = 2, m_3 = \frac{8}{3}$,and $x - 4 = -\frac{8}{3}, -2, -\frac{4}{3}$,
and values of $f(x) = \frac{1}{x-4}$ at the midpoints are: $-\frac{3}{8}, -\frac{1}{2}, -\frac{3}{4}$ and
$M_3 = \frac{2}{3}\left[-\frac{3}{8} - \frac{1}{2} - \frac{3}{4}\right] = \frac{2}{3}\cdot -\frac{3+4+6}{8} = -\frac{13}{12}.$
Now,we can evaluate the integral directly:$[\ln|x-4|]_1^3 = \ln 1 - \ln 3 = -\ln 3,$

so $\ln 3 \doteq \frac{13}{12} = 1.083$

For 2)

y-axis→ "shells"→ dx

we need two lines first,through $(1,1)$ and $(2,0)$: $y = 2 - x$......top

through $(1,-2)$ and $(2,0)$: $y = 2x - 4$.....bottom

Radius of the cylindrical shell is x,the height

$H =$ "top–bottom"$=(2-x)-(2x-4) = 6-3x = 3(2-x)$

so $V = 2\pi \int\limits_a^b RH dx = 2\pi \int\limits_1^2 x \cdot 3(2-x)\, dx = 6\pi \left[x^2\right]_1^2 - 2\pi \left[x^3\right]_1^2 =$

$= 6\pi[4-1] - 2\pi[8-1] = 4\pi.$

For 3)

For $\quad y^3 = x^2$ between points O$(0,0)$ and $P(8,4)$

$y = x^{\frac{2}{3}}$ and $) \le x \le 8 \qquad y' = \frac{2}{3}x^{-\frac{1}{3}}$

$1 + (y')^2 = 1 + \dfrac{4}{9x^{\frac{2}{3}}} = \dfrac{9x^{\frac{2}{3}}+4}{9x^{\frac{2}{3}}}$ then

$s = \int\limits_0^8 \sqrt{1+(y')^2}dx = \int\limits_0^8 \sqrt{\dfrac{9x^{\frac{2}{3}}+4}{9x^{\frac{2}{3}}}}dx = \int\limits_0^8 \dfrac{\sqrt{9x^{\frac{2}{3}}+4}}{3x^{\frac{1}{3}}}dx$

subst.$u = 9x^{\frac{2}{3}} + 4 \qquad du = 18\frac{1}{3x^{\frac{1}{3}}}dx$

x	u
8	40
0	4

$s = \frac{1}{18}\int\limits_4^{40} \sqrt{u}du = \frac{1}{18}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_4^{40} = \frac{1}{27}\left[40\sqrt{40} - 4\sqrt{4}\right] = \frac{8}{27}\left[10\sqrt{10}-1\right].$

C **For 1)**

For $\int\limits_1^3 \dfrac{1}{x}\, dx$ and $n = 4$

the length of the subintervals is $\triangle x = h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$

the points of partition are:

$a = x_0 = 1, x_1 = \frac{3}{2}, x_2 = 2, x_3 = \frac{5}{2}, b = x_4 = 3$, then midpoints are:

$m_1 = \frac{5}{4}, m_2 = \frac{7}{4}, m_3 = \frac{9}{4}, m_4 = \frac{11}{4}$,and the values of $f(x) = \frac{1}{x}$ are: $\frac{4}{5}, \frac{4}{7}, \frac{4}{9}, \frac{4}{11}$

So $M_4 = \frac{1}{2}\left[\frac{4}{5}+\frac{4}{7}+\frac{4}{9}+\frac{4}{11}\right] = 2\left[\frac{16}{55}+\frac{16}{7\cdot 9}\right] = 32\left[\frac{1}{55}+\frac{1}{63}\right] = 1.0897547$

$\int\limits_1^3 \dfrac{1}{x}\, dx = [\ln x]_1^3 = \ln 3 = 1.0986123$,so the error is 0.0088576

since error=exact value –approximation

For 2)

First,find the intersection of the parabola and line $2 - x^2 = x, x > 0$,so

$0 = x^2 + x - 2 = (x+2)(x-1) \qquad x = 1, y = 1$

parabola ...top,line...bottom,$0 \le x \le 1$

x-axis→ "washers"→dx

$\backslash V = \pi \int\limits_0^1 \left(top^2 - bottom^2\right) dx = \pi \int\limits_0^1 \left[\left(2-x^2\right)^2 - x^2\right] dx =$

$$= \pi \int_0^1 \left[4 - 4x^2 + x^4 - x^2\right] dx = \pi \int_0^1 \left[4 - 5x^2 + x^4\right] dx =$$

$$= \pi \left[4x - \tfrac{5}{3}x^3 + \tfrac{1}{5}x^5\right]_0^1 = \pi \left[4 - \tfrac{5}{3} + \tfrac{1}{5}\right] = \frac{38}{15}\pi.$$

For 3)

For $x^2 + y^2 = 5$ between points $Q\,(2,1)$ and $P\,(1,2)$

$y = \sqrt{5 - x^2} \qquad 1 \le x \le 2 \qquad y' = \dfrac{-x}{\sqrt{5 - x^2}}$

$1 + (y')^2 = 1 + \dfrac{x^2}{5 - x^2} = \dfrac{5}{5 - x^2}$ then

$$s = \int_1^2 \sqrt{1 + (y')^2}dx = \sqrt{5}\int_1^2 \frac{1}{\sqrt{5 - x^2}}dx = \sqrt{5}\left[\arcsin \tfrac{x}{\sqrt{5}}\right]_1^2 =$$

$$= \sqrt{5}\left[\arcsin \tfrac{2}{\sqrt{5}} - \arcsin \tfrac{1}{\sqrt{5}}\right].$$

D For 1)

For $\displaystyle\int_1^2 \frac{1}{x^2}\, dx$ and $n = 3$.

the length of the subintervals is $\triangle x = h = \frac{b-a}{n} = \frac{2-1}{3} = \frac{1}{3}$, so the points of partition are:
$a = x_0 = 1, x_1 = \frac{4}{3}, x_2 = \frac{5}{3}, b = x_3 = 2$, then the values of $f(x) = \frac{1}{x^2}$
$y_0 = 1, y_1 = \frac{9}{16}, y_2 = \frac{9}{25}, y_3 = \frac{1}{4}$,
So $T_3 = \frac{1}{3}\left[\frac{1}{2} + \frac{9}{16} + \frac{9}{25} + \frac{1}{8}\right] = \frac{1}{3}\left[9 \cdot \frac{41}{400} + \frac{5}{8}\right] = \frac{123}{400} + \frac{5}{24} = \frac{619}{1200} = 0.51583$
By direct integration:

$\displaystyle\int_1^2 \frac{1}{x^2}\, dx = \left[\frac{-1}{x}\right]_1^2 = \frac{-1}{2} + 1 = \frac{1}{2}$,so the error is 0.01583.

For 2)

First,sketch the region :hyperbola $y = \frac{2}{x}$...bottom;line $y = 4$.. top
$\frac{1}{2} \le x \le 1$
x-axis$\rightarrow$ " washers"$\rightarrow dx$

$$V = \pi \int_{\frac{1}{2}}^1 \left[(top)^2 - (bottom)^2\right] dx = \pi \int_{\frac{1}{2}}^1 \left[4^2 - \left(\tfrac{2}{x}\right)^2\right] dx =$$

$$= 16\pi \cdot \left(1 - \tfrac{1}{2}\right) - \pi \int_{\frac{1}{2}}^1 4x^{-2}dx = 8\pi - 4\pi\left[-\tfrac{1}{x}\right]_{\frac{1}{2}}^1 = 8\pi + 4\pi\,(1 - 2) = 4\pi.$$

For 3)

For $y^2 = x^3$ between points O$(0,0)$ and $P\,(4,-8)$

$y = -x^{\frac{3}{2}} \qquad 0 \le x \le 4 \qquad y = -\frac{3}{2}\sqrt{x}$

$1 + (y')^2 = 1 + \frac{9}{4}x = \dfrac{4 + 9x}{4}$ then

$$s = \int_0^4 \sqrt{1 + (y')^2}dx = \tfrac{1}{2}\int_0^4 \sqrt{4 + 9x}dx = \tfrac{1}{2}\left[\tfrac{2}{3}\frac{(4 + 9x)^{\frac{3}{2}}}{9}\right]_4^{40}$$

also by subst.$u = 4x + 9$

$$s = \tfrac{1}{27}\left[40^{\frac{3}{2}} - 4^{\frac{3}{2}}\right] = \tfrac{4^{\frac{3}{2}}}{27}\left[10^{\frac{3}{2}} - 1\right] = \tfrac{8}{27}\left[10\sqrt{10} - 1\right].$$