

1. Calculate

$$\begin{aligned} \int \frac{\sqrt{x} + \ln(x)}{x} dx &= \int \frac{1}{\sqrt{x}} dx + \int \frac{\ln(x)}{x} dx \Big|_{\substack{u=\ln(x) \\ du=\frac{1}{x} dx}} = \\ &= 2\sqrt{x} + \int u du = 2\sqrt{x} + \frac{1}{2}u^2 + C = 2\sqrt{x} + \frac{1}{2}[\ln(x)]^2 + C. \end{aligned}$$

2. Calculate

$$\begin{aligned} \int_0^\pi \cos^4(p) dp &= \int_0^\pi [\cos^2(p)]^2 dp = \int_0^\pi \left[\frac{1 + \cos(2p)}{2} \right]^2 dp = \\ &= \frac{1}{4} \int_0^\pi [1 + 2\cos(2p) + \cos^2(2p)] dp = \frac{1}{4} [p]_0^\pi + \sin(2p) \Big|_0^\pi + \int_0^\pi \frac{1 + \cos(4p)}{2} dp = \\ &= \frac{1}{4} \left[\pi + \frac{1}{2} \left(p + \frac{1}{4} \sin(4p) \right) \Big|_0^\pi \right] = \frac{1}{4} \left[\pi + \frac{\pi}{2} \right] \end{aligned}$$

3. Calculate

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \tan^3(r) dr &= \int_0^{\frac{\pi}{4}} \tan(r) \tan^2(r) dr = \int_0^{\frac{\pi}{4}} \tan(r) [\sec^2(r) - 1] dr = \\ &= \int_0^{\frac{\pi}{4}} \tan(r) \sec^2(r) dr \Big|_{\substack{u=\tan(r) \\ du=\sec^2(r) dr}} - \int_0^{\frac{\pi}{4}} \tan(r) (-1) dr = \\ &= \int_0^1 u du - \ln |\cos(r)| \Big|_0^{\frac{\pi}{4}} = \frac{1}{2} - \ln\left(\frac{1}{\sqrt{2}}\right). \end{aligned}$$