Solutions to Lab Quiz 1, Math 253 B27/B28 Friday 1/21, 10:00

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1. Calculate

$$\int \frac{x - \sin(2x)}{x^2 + \cos(2x)}\,dx\,\Big|_{\substack{u=x^2+\cos(2x)\\ du=2-2\sin(2x)}} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+\cos(2x)| + C.$$

2. Calculate

$$\int_0^{\frac{\pi}{2}} \cos^5(u)\sin^3(u)\,du = \int_0^{\frac{\pi}{2}} \cos^5(u)\sin^2(u)\ \sin(u)\,du\Big|_{\substack{v=\cos(u)\\ dv=-\sin(u)du}} =$$

$$-\int_1^0 v^5[1-v^2]dv.$$

Now switch the limits and change the sign of the integral:

$$= \int_0^1 v^5[1-v^2]\,dv = [\frac{v^6}{6} - \frac{v^8}{8}]\,|_0^1 = \frac{1}{24}.$$

Can also be done by splitting off a single power of $\cos(u)$ and using the substitution $v = \cos(u),\ dv = -\sin(u)du$.

3. Calculate

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^4(r)\,dr = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2(r)\tan^2(r)\,dr =$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2(r)[\sec^2(r)-1]\,dr = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2(r)\sec^2(r)\,dr\Big|_{\substack{u=\tan(r)\\ du=\sec^2(r)dr}} + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2(r)[-1]\,dr =$$

$$= \int_{-1}^{1} u^2\,du - \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} [\sec^2(r) - 1]\,dr =$$

$$= \frac{u^3}{3}\,|_{-1}^{1} - [\tan(r) - r]\,|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{2}{3} - [2 - \frac{\pi}{2}].$$