Lab Quiz 2 Solutions, Math 253 B25/B26 **Wed. 02/2 , 1300 hours**

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1. Calculate $\int \frac{1}{5+x^2+2x}\,dx$.. We complete the square: $5 + x^2 + 2x = 4 + (x+1)^2$. So we can write:

$$\int \frac{1}{5+x^2+2x}\,dx = \int \frac{1}{4+(x+1)^2}\,dx =$$
$$= \frac{1}{4}\int \frac{1}{1+\left(\frac{x+1}{2}\right)^2}\,dx = \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C.$$

2. (a) Simplify $\tan\left(\arcsin\left(\frac{1}{5}\right)\right)$. If we define $\theta = \arcsin\left(\frac{1}{5}\right)$, then we can draw a right triangle with θ at one vertex and side opposite of 1, hypotenuse of 5. This makes the side adjacent to θ of length $\sqrt{24}$ by Pythagoras. Thus the tangent of θ is $\frac{1}{24}$.
(b) Calculate

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx\,\Big|_{\substack{u=x^2\\ du=2x\,dx}} = \int \frac{1}{\sqrt{1-u^2}}\,du =$$
$$= \arcsin(u) + C = \arcsin\left(x^2\right) + C.$$

3. Given $f(x) = \arcsin(x-2)$, (a) find the domain and range of f; (b) show f is one-to-one; (c) find a formula for f^{-1}, the inverse of f.

(a) The domain of $\arcsin(u)$ is $-1 \le u \le +1$, so we must have $-1 \le x-2 \le +1$, i.e., $1 \le x \le 3$. The range of f is the range of $\arcsin(u)$ for $-1 \le u \le +1$, i.e., $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
(b) The derivative of f is

$$f'(x) = \frac{1}{\sqrt{1-(x-2)^2}} > 0,$$

so f is one-to-one. It is also correct to observe that the graph of f is just the graph of $\arcsin(u)$ with the $y-axis$ shifted 2 units to the right, so f passes the horizontal line test.
(c) If $y = \arcsin(x-2)$ then $x = 2+\sin(y)$. Interchanging x and y , $f^{-1}(x) = 2+\sin(x)$.